Mathematical Tables
and other
Aids to Computation

A Quarterly Journal edited on behalf of the
Committee on Mathematical Tables
and Other Aids to Computation
by

RAYMOND CLARE ARCHIBALD
DERRICK HENRY LEHMER

WITH THE COÖPERATION OF

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SOLOMON ACHILLOVICH JOFFE

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COMMITTEE ON MATHEMATICAL TABLES AND AIDS TO COMPUTATION

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Introductory

This Quarterly Journal, a new publication of the National Research Council, is to serve as a clearing-house for information concerning mathematical tables and other aids to computation. Especially during the past decade have tools for computation been vastly multiplied. These tools, or accounts of them, are to be found in an enormous international range of book, pamphlet, and periodical publication, not only in the fields of Pure Mathematics, Physics, Statistics, Astronomy, and Navigation, but also in such fields as Chemistry, Engineering, Geodesy, Geology, Physiology, Economics, and Psychology. An attempt will here be made to guide varied types of inquirers to such material. This guidance may assume diverse forms. One of these might be by a bibliographic article, dealing in a given field with a very special group of tables; an example of this type appears below (RMT **81**) where there is a survey of tables of trigonometric functions with radian arguments. A longer guiding article might, in a much larger portion of a field, survey the most useful tables for current research. Illustrations of this type are the articles on mathematical tables, by James Henderson, in the fourteenth edition (1929) of the *Encyclopædia Britannica*, and by L. J. Comrie in Royal Astronomical So., *Monthly Notices*, v. 92, 1932, p. 339–347, supplemented by British Astronomical Association, *Handbook for 1929*, p. 38–43. Among the numerous critical reviews of recent tables may sometimes be a table, prepared primarily for use in one field, which turns out to be of value to a worker in an entirely different field. So far as practicable, the Editor will seek to have both articles and reviews written partly in non-technical language so that scholars in all fields may from such material occasionally glean something of personal advantage.

The regular section on the description and location of Unpublished Mathematical Tables ought not only to be a suggestive aid in research, but also to prevent useless duplication of costly effort. The publication in this Journal of shorter new tables of importance may be later considered advisable. A section devoted to Queries, and Replies to Queries is to make possible a general appeal for information not otherwise procurable. And further, after the first issue the material for guidance will constantly include items of interest concerning calculating machines and the types of problems they can solve.

With this first number of the Quarterly now before you, such general remarks will suffice. It may be well, however, to add notes, not to be found elsewhere, concerning certain editorial decisions and notations. From time to time it may be desirable to refer to, and to discuss, an out-of-print and scarce publication. Since it is always dangerous for the writer of an article to make statements about a work not personally inspected, it will be editorial policy to place a special sign "o" before every title of material unseen by the author or Editor, either in the original or in film form. In the case of a rare book discussed it is also planned to indicate, if possible, a library of this country or Great Britain where this book may be seen.

Those who have worked much with mathematical tables are doubtless familiar with the admirable work of James Henderson, *Bibliotheca Tabularum Mathematicarum being a Descriptive Catalogue of Mathematical Tables . . . Part I, Logarithmic Tables* (A. *Logarithms of Numbers*) (*Tracts for Computers*, no.

XIII), Cambridge, University Press, 1926, iv, 2 plates, 208 p. This was one of the series of *Tracts*, founded and edited by Karl Pearson (1857–1936). In preparing the volume Mr. Henderson gathered together considerable additional material for planned later volumes (never published), especially the one on trigonometric functions. All of these manuscripts were in February 1937, most kindly loaned to our Committee, by Karl Pearson's son, Professor E. S. Pearson, of the University of London, and Mr. Henderson. In the present issue of our Journal as in later numbers, a reference to "Henderson" will indicate that use has been made of his material, which was almost wholly prepared with the volumes under discussion in his hands.

In editing, a distinction will be made between the abbreviation for the logarithm of a number to base 10 and to base e; the first is $\log N$, and the second $\ln N$. Such notations as 8D, 8S, will be found useful for "8 places of decimals," "8 significant figures." In transliteration from Russian the system used by the Library of Congress will be employed. This is set forth in G. F. v. Ostermann and A. E. Giegengack, *Manual of Foreign Languages for the Use of Printers and Translators*. Third rev. and enl. ed., Washington, D. C., 1936.

On covers 2–3 of each number of our Quarterly are to be found the names and addresses of members of our committee, as well as the Classifications **A, B,** · · · , **Z,** adopted for the preparation of our Reports. Communication from members of our Executive Committee will be signed with initials only. The Classifications will be frequently introduced, especially in the review section, to indicate the general nature of the contents of an article, pamphlet, or volume. For example [**A, B, C, D, F, M**], after a title would mean that the publication contained tables under the following headings: arithmetic, powers, logarithms, circular functions, theory of numbers, and integrals. Reviews of new tables appear under the heading Recent Mathematical Tables and are numbered consecutively. A convenient reference notation for a particular review is, therefore, RMT **75**. So also MTE **5** for an item under Mathematical Tables—Errata, MAC **4** for an entry under Mechanical Aids to Computation, and Q **6** or QR **6** for Query 6 or Reply to Query **6**. To assist in familiarizing the reader with such abbreviations we shall, for a time, add them to the Contents on cover 4.

Up to the present the chief duty of the Committee has been to prepare a series of comprehensive Reports on mathematical tables, valuable for various types of research in different fields. D. H. Lehmer's *Guide to Tables in the Theory of Numbers* was published in February, 1941. It is expected that a more elaborate Report, in another field, may be ready for publication during 1943. Since most members of the Committee are at the present time deeply involved in national service, the completion of still further Reports in the near future is likely to be very difficult to achieve.

Meanwhile, it is hoped this new periodical may render notable current service, and that in years to come it, and the Reports, may be regarded as the standard sources to which one may naturally turn for guidance in connection with all mathematical tables of importance in contemporary research.

On behalf of the Committee,

R. C. A.

R. C. A. greatly regrets the apparent necessity for numerous personal contributions in this issue, as well as in the second. It seems certain that elimination in this regard shall be noticeably operative in the third and later issues.

RECENT MATHEMATICAL TABLES

Under this heading Tables published currently, and within the past ten years, will be reviewed. During the years 1934–1936 twelve articles on "Mathematical Tables" appeared in the quarterly journal, *Scripta Mathematica*, New York, v. 2–4. These articles included reviews of tables, numbered consecutively 1 to 74. Since it is not planned that any of these tables shall be reviewed here, a list of them is assembled below. The new series of articles is to assume quite a different character from the earlier series in that the reviews will be longer and no notice will be taken of certain types of elementary tables previously considered. In spite of this, it has been suggested by more than one correspondent that convenience of reference would be enhanced by having the new series of reviews numbered consecutively, beginning with number 75. Hence this is to be our plan, and we list below the publications reviewed in numbers 1–74.

The previous reviews appeared in *Scripta Mathematica* as follows: v. 2, 1934 (nos. 1–12, p. 91–93, 297; nos. 13–24, p. 193–197; nos. 25–32, p. 297–299; nos. 33–39, p. 379–380); v. 3, 1935 (nos. 40–41, p. 97–98; nos. 42–44, p. 192–193; nos. 45–48, p. 282–283; nos. 49–52, p. 364–366); v. 4, 1936 (nos. 53–58, p. 101–104; nos. 59–65, p. 198–201; nos. 66–70, p. 294–295; nos. 71–74, p. 338–340).

1. *Noordhoff's Schooltafel*, Groningen, P. Noordhoff, 1933, 112 p. 14.9×23.6 cm.
2. W. L. Hart, compiler, *Logarithmic and Trigonometric Tables*, Boston, New York, and Chicago, D. C. Heath and Co., [1933], iv, 124 p. 15×22 cm.
3. R. S. Burington, *Handbook of Mathematical Tables and Formulas*, Sandusky, Ohio, Handbook Publishers, Inc., 1933, 8, 251 p. 13.2×19.7 cm.
4. P. Herget, "A table of sines and cosines to eight decimal places," *Astr. Jn.*, v. 42, 28 Jan. 1933, cols. 123–125. 23.7×31.3 cm.
5. B. V. Numerov, *Tablit͡sy natural'nykh znacheniĭ trigonometricheskikh funkt͡siĭ s 5-'i͡u desi͡atichnymi znakami* [Tables of the natural values of the trigonometric functions to five places of decimals], Leningrad-Moscow, Gosudarstvennoe Tekhniko-teoreticheskoe Izdatel'stvo, 1933, 58 p. 16.9×25.2 cm.
6. B. V. Numerov, *Tablit͡sy dli͡a vychisleni͡ia geograficheskikh i pri͡amougol'nykh koordinat Gaussa-Kriugera, dli͡a shirot ot 36° do 72° s tochnost'i͡u do 0.1 metra i 0''.01 (dli͡a raboty s arifmometrom). Tables for calculation of geographic and rectangular coordinates of Gauss-Krüger for latitudes from 36° to 72° with accuracy to 0.1 m. and 0''.01 (for work with a calculating machine)*. Leningrad, Astronomical Institute, 1933, 80 p. 17.5×26.1 cm.
7. J. Plassmann, *Tafel der Viertel-Quadrate aller Zahlen von 1 bis 20009, zur Erleichterung des Multiplizierens vierstelliger Zahlen. Mit vielen Ratschlägen für das praktische Rechnen in Handel, Gewerbe and Wissenschaft.* Leipzig, Max Jänecke, 1933, 26, 200 p. 15×21.6 cm.
8. F. J. Duarte, *Nouvelles Tables Logarithmiques à 36 décimales*, Paris, Gauthier-Villars, 1933, xxviii, 128 p. 17.1×25.5 cm.
9. J. Boccardi, *Tables logarithmiques des factorielles jusqu'à 10000!*, Cavaillon, 1932, 36 p. +1 sheet errata. 14×20.9 cm.

10. H. TALLQUIST, "Tafel der 24 ersten Kugelfunktionen $P_n(\cos \theta)$," Societas Scientiarum Fennica, *Commentationes Physico-Mathematicae*, v. 6, no. 3, Mar. 1932, 11 p. The title page for volume 6 is dated 1933. 16.5×23.5 cm.
11. A. DINNIK, "Tafeln der Besselschen Funktionen von der gebrochenen Ordnung" [title also in Ukrainian], Allukrainische Akademie der Wissenschaften, *Naturwissenschaftlich-technische Klasse*, 1933, 29 p. 17.3×26.5 cm.
12. K. HAYASHI, *Tafeln für die Differenzenrechnung sowie für die Hyperbel-, Besselschen, elliptischen und anderen Funktionen*, Berlin, Springer, 1933. 21×27.7 cm.
13. A. GÉRARDIN, (a) "Liste inédite des 1001 nombres premiers de la forme $2x^2+2x+1$ pour x compris entre 15,800 et 23,239, $f(x)$ de 499,311,601 à 1,008,148,721"; (b) "Liste inédite de 1035 nombres premiers de 8 et 9 chiffres extraits de diverses séries quadratiques," *Sphinx Oedipe*, v. 27, May 1932, 4, 4 p. 13.8×22.1 cm.
14. M. KRAITCHIK and S. HOPPENOT, "Les grands nombres premiers," *Sphinx*, v. 3, Oct. 1933, p. 145–146; Nov. 1933, p. 161–162. 15.5×24 cm.
15. H. W. WEIGEL, *$x^n+y^n=z^n$? Die elementare Lösung des Fermat-Problems . . .*, Leipzig, [1933?]; preface dated November, 1932. 45.3×28.7 cm.
16. *Minimum Decompositions into Fifth Powers*, prepared under the direction of L. E. Dickson (Br. Assoc. Adv. Sci., *Mathematical Tables*, v. 3) London, 1933, 4to, 8, 368 unnumbered pages. 21.3×27.8 cm.
17. N. G. W. H. BEEGER, *Additions and corrections to "Binomial Factorisation" by . . . A. J. C. Cunningham . . . 1923–29*, Amsterdam, 1933, [ii], 12 p. Hectograph print by the author at Nicolaas Witsenkade 10. 17×21 cm.
18. J. SHERMAN, "A four place table of $(\sin x)/x$," *Z.f.Kristallographie*, Leipzig, v. A85, p. 404–419, June 1933. 15.3×23 cm.
19. A. UMANSKY, "$A=\cosh \xi \cos \xi$, $B=\frac{1}{2}(\cosh \xi \sin \xi+\sinh \xi \cos \xi)$, $C=\frac{1}{2}\sinh \xi \sin \xi$, $D=\frac{1}{4}(\cosh \xi \sin \xi-\sinh \xi \cos \xi)$; Tafeln hyperbolisch-trigonometrischer Funktionen" [title also in Ukrainian], Académie des Sciences d'Ukraine, *Classe des sciences naturelles et techniques, Jn. du Cycle Industriel et Technique*, Kiev, nos. 2–3, 1932, p. 97–117 of Umansky's paper. 18×25.5 cm.
20. J. B. RUSSELL, "A table of Hermite functions," *Jn. of Math. and Physics*, Mass. Inst. Tech., v. 12, May 1933, p. 291–297. 17×25.2 cm.
21. G. PRÉVOST, *Tables de Fonctions Sphériques et de leurs Intégrales pour calculer les coefficients du développement en série de Polynomes de Laplace d'une fonction de deux variables indépendantes*. Bordeaux and Paris, 1933, xxxii, 96, viii*, 163* p. 22×27.5 cm.
22. E. L. INCE, (a) "Tables of the elliptic-cylinder functions"; (b) "Zeros and turning points of the elliptic-cylinder functions," Royal So. Edinburgh, *Proc.*, v. 52, 1932, (a) p. 355–423; (b) p. 424–433. 17.3×25.5 cm.
23. A. J. THOMPSON, *Logarithmetica Britannica being a Standard Table of Logarithms to Twenty Decimal Places . . . Part VI . . . Numbers 60,000 to 70,000*. (*Tracts for Computers*, no. XVIII.) London, Cambridge Univ. Press, 1933, 102 p.+a plate. 22×27.7 cm.
24. *Tables of the Higher Mathematical Functions*, computed and compiled under the direction of H. T. Davis with the cooperation of 20 computers and assistants. Bloomington, Ind., v. 1, 1933, 14, 377 p.+2 plates. 17.5×24.9 cm.
25. E. W. BROWN and D. BROUWER, *Tables for the Development of the Disturbing Function with Schedules for Harmonic Analysis*, Cambridge, Engl.,

1933, 2+p. 69–157. Reprinted from Yale University Observatory, *Trans.*, v. 6, pt. 5. 23.5×31.8 cm.

26. H. J. LUCKERT, *Über die Integration der Differentialgleichung einer Gleitschicht in zäher Flüssigkeit* (Diss. Berlin), Leipzig, 1933; reprint from Berlin, Universität, Institut f. angewandte Mathem., *Schriften d. mathem. Seminars*, v. 1, p. 245–274. 16×24 cm.
27. L. BENDERSKY, "Sur la fonction gamma généralisée," *Acta Math.*, v. 61, Nov. 1933. 22×29.5 cm.
28. P. HARZER, "Tabellen für alle statistischen Zwecke in Wissenschaft und Praxis, insbesondere zur Ableitung des Korrelations-Koeffizienten und seines mittleren Fehlers," Bayer. Akad. d. Wiss., *Abh., math.-natw. Abt.*, neue Folge, Heft 21, 1933, iv, 91 p. 23×28.5 cm.
29. A. S. PERCIVAL, *Mathematical Facts and Formulae*, London and Glasgow, Blackie, 1933, vi, 125 p. 12×18.3 cm.
30. H. B. DWIGHT, *Tables of Integrals and Other Mathematical Data*, New York, Macmillan, 1934, x, 222 p. 14.4×21.5 cm.
31. F. E. FOWLE, ed., *Smithsonian Physical Tables*, eighth rev. ed., Washington, D. C., 1933, liv+682 p. 15.3×23.1 cm.
32. E. JAHNKE and F. EMDE, *Tables of Functions with Formulae and Curves. Funktionentafeln mit Formeln und Kurven.* Second rev. ed., with 171 figures. Leipzig, Teubner, 1933, xviii, 330 p. 16×24 cm.
33. W. J. SEELEY, *Table for the Rapid Evaluation of the Square-Root-of-the-Sum-of-the-Squares of two Numbers*, Published by the author, Duke Univ., Durham, N. C., 1933, 8 unnumbered p. rotograph print. 13.5 ×20.3 cm.
34. D. KATZ, *Pocket Tables for Cubics. A Systematic Method for Algebraic Treatment of Cubic Equations*, South Milwaukee, Wis., 1933, 6 p. in folded sheet 11×8¾ in.
35. M. KRAÏTCHIK and S. HOPPENOT, "Les grands nombres premiers," *Sphinx*, v. 4, June 1934, p. 81–82. 15.5×24 cm.
36. M. KRAÏTCHIK, "Les grands nombres premiers," *Mathematica*, Cluj, v. 7, 1933, p. 92–94. 17×24.1 cm.
37. H. T. DAVIS, "Polynomial approximation by the method of least squares," *Annals of Math. Statistics*, v. 4, Aug. 1933, p. 155–195. 17.5×25.2 cm.
38. F. W. SPARKS, *Universal Quadratic Zero Forms in Four Variables*, Chicago, Ill., private edition distributed by the Univ. of Chicago libraries, 1933. Facsimile prints, except for cover and title page. 17×24 cm.
39. E. McC. CHANDLER, *Waring's Theorem for Fourth Powers*. Chicago, Ill., private edition distributed by the Univ. of Chicago libraries, 1933, iv, 62 p. Pages ii and 1–62 are facsimile prints. 17×24 cm.
40. M. KRAÏTCHIK and S. HOPPENOT, "Les grands nombres premiers," *Sphinx*, v. 4, Aug. 1934, p. 113–114. 15.5×24 cm.
41. E. L. INCE, *Cycles of Reduced Ideals in Quadratic Fields*. (Br. Ass. Adv. Sci., *Mathematical Tables*, v. 4.) London, 1934, 4to, xvi, 80 p. 20.8×28 cm.
42. A. J. THOMPSON, *Logarithmetica Britannica being a standard Table of Logarithms to Twenty Decimal Places . . . Part I . . . Numbers 10,000 to 20,000.* (*Tracts for Computers*, no. 19.) Cambridge, Univ. Press, 1934, 107 p. (unnumbered)+2 photogravure plates giving a facsimile of the will of Henry Briggs. 22×27.7 cm.
43. P. POULET, "De nouveaux amicables," *Sphinx*, v. 4, Sept. 1934, p. 134–135. 15.5×24 cm.
44. M. KRAÏTCHIK and S. HOPPENOT, "Les grands nombres premiers," *Sphinx*, v. 4, Nov. 1934, p. 161–162. 15.5×24 cm.

45. Astronomicheskiĭ Institut, *Ephemerides for the Determination of Time-corrections by equal Altitudes (Zinger's method) for 1935*, Leningrad, 1934. 17.2×26.2 cm.

46. J. BOUMAN and W. F. DE JONG, "Grafische bepaling van buigingsfiguren," Akad. v. Wetenschappen, Amsterdam, *Verhandelingen, afd. Natuurkunde*, v. 4, no. 4, 1931. 17.5×25.5 cm.

47. O. LOHSE, *Tafeln für numerisches Rechnen mit Maschinen*. Zweite Auflage neubearbeitet von P. V. Neugebauer. Leipzig, Engelmann, 1935. vi, 113 p. 17×24.2 cm.

48. S. HOPPENOT, *Table des Solutions de la Congruence* $x^4 \equiv -1$ (mod. N) *pour* $100000 < N < 200000$, Brussels, Librairie du "Sphinx," 1935, 18 p. 15.6×24.1 cm.

49. E. S. ALLEN, *Six-Place Tables. A Selection of Tables of Squares, Cubes, Square Roots, Cube Roots, Fifth Roots and Powers, Circumferences and Areas of Circles, Common Logarithms of Numbers and of the Trigonometric Functions, the Natural Trigonometric Functions, Natural Logarithms, Exponential and Hyperbolic Functions and Integrals*. Fifth edition. New York and London, 1935, xxiii, 175 p. 11×18 cm.

50. N. SAMOILOVA-I͡AKHONTOVA, *Tablit͡sy Ellipticheskikh Integralov* [Tables of Elliptic Integrals]. Moscow and Leningrad, 1935, 108 p.+errata slip. 17×24.5 cm.

51. A. M. LEGENDRE, *Tables of the Complete and Incomplete Elliptic Integrals, reissued from Tome II of Legendre's Traité des Fonctions Elliptiques, Paris, 1825, with an Introduction* by Karl Pearson. *With autographed Portrait of Legendre*. Cambridge, University Press, 1934, xliii, 94 p. 20.5×25.5 cm.

52. H. GUPTA, "Decompositions into squares of primes," Indian Acad. Sci. *Proc., s.A*, v. 1, p. 789–794, May 1935. 18×24.6 cm.

53. J. R. AIREY, "The circular and hyperbolic functions, argument $x/\sqrt{2}$," *Phil. Mag.*, s. 7, v. 20, Oct. 1935, p. 721–731. 17×25.3 cm.

54. J. R. AIREY, "The circular and cosine functions, argument $\log_e x$," *Phil. Mag.*, s. 7, v. 20, Oct. 1935, p. 731–738. 17×25.3 cm.

55. L. E. DICKSON, *Researches on Waring's Problem* (Carnegie Institution of Washington, *Publication* no. 464), Washington, 1935, v, 257 p. 17×25 cm.

56. R. C. SHOOK, *Concerning Waring's Problem for Sixth Powers*. Diss., Univ. Chicago, Chicago, Ill., 1934, iv, 38 p. 17×24 cm.

57. B. W. JONES, *A Table of Eisenstein-reduced Positive Ternary-Quadratic Forms of Determinant* $\leqq 200$ (National Research Council, *Bulletin*, no. 97), Washington, National Acad. Sci., 1935, 51 p. 17×24.5 cm.

58. H. ROUSSILHE and BRANDICOURT, *8 Place Tables of the Natural Values of Sines, Cosines and Tangents according to the Centesimal System, for each Centigrade from 0 to 100 Grades . . . followed by 20 Place Tables of the Natural Values of the Six Trigonometrical Functions according to the Centesimal System for each Grade from 0 to 100 Grades. Taken from the Tables of M. Andoyer*, New revised edition (International Geodetic and Geophysical Union Association of Geodesy, *Special Publication*, no. 1), Paris, 1933, 23, 99, 18 p. 18×26.7 cm.

59. V. R. BURSIAN, "Tablit͡sy znacheniĭ funkt͡sii $I_{1/3}$" [Tables of values of the function $I_{1/3}$], Leningradskiĭ Gosudarstvennyĭ Universitet imeni A. S. Bubnova, *Uchenyi͡e Zapiski, . . . serii͡a Fizicheskikh Nauk* [Leningrad State University in the name of A. S. Bubnov. Annals, . . . series of

physical sciences], Leningrad, v. 1, no. 1, 1935, p. 4–9; German abstract, p. 8–9. 22.5×29 cm.

60. F. Triebel, *Rechen-Resultate. Tabellen zum Ablesen der Resultate von Multiplikationen und Divisionen bis 100×1 000=100 000 in Bruchteilen und ganzen Zahlen. Zum praktischen Gebrauch für Stückzahl-, Lohn-, und Prozentberechnungen. Rechenhilfsmittel für alle Arten des Rechnens mit Zahlen jeder Grösse. Radizieren (Wurzelsuchen) nach vereinfachtem Verfahren.* Sixth ed. Berlin, M. Krayn, 1934, 285 p. 19×26.5 cm.

61. L. Silberstein, "On complex primes," *Phil. Mag.*, s. 7, v. 19, June 1935, p. 1097–1107. 17×25.3 cm.

62. J. Peters, A. Lodge and E. J. Ternouth, E. Gifford, *Factor Table giving the Complete Decomposition of all Numbers less than 100,000.* (British Association for the Advancement of Science, *Mathematical Tables*, v. 5.) London, B. A. A. S., 1935, xv, 291 p. 22.5×28 cm.

63. J. R. Airey, "Toroidal functions and complete elliptic integrals," *Phil. Mag.*, s. 7, v. 19, Jan. 1935, p. 177–188. 17×25.3 cm.

64. J. P. Möller and H. Q. Rasmusen, "Tafel der Funktion $x^{2/3}$ zur Verwendung bei parabolischer Bahnbestimmung nach der Methode von B. Strömgren," *Astron. Nach.*, v. 258, 4 Jan. 1936, cols. 9–10.

65. A. J. Thompson, *Logarithmetica Britannica being a standard Table of Logarithms to Twenty Decimal Places . . . Part VII . . . Numbers 70,000 to 80,000.* (*Tracts for Computers*, no. 20.) Cambridge, University Press, 1935, 106 p. (unnumbered)+3 p. of facsimiles. 22×28 cm.

66. S. Sakamoto, *Tables of Gudermannian Angles and Hyperbolic Functions.* Tokyo, 1934, 157 p. 12.6×18.6 cm.

67. L. I͡a. Neĭshuler, *Tablit͡sy priblizhennykh Vychisleniĭ: Delenie, Umnozhenie, desi͡atichnye i naturalnye logarifmy, polnye Kvadraty chetyrekhznachnykh Chisel* [Tables of approximate Computations: Division, Multiplication, decimal and natural Logarithms, complete Squares of four-place Numbers], second edition enlarged, Moscow and Leningrad, 1933, 139 p. 19.6×26.3 cm.+1 plate 66×43.5 cm.

68. L. I͡a. Neĭshuler, *Tablit͡sy Proizvedeniĭ Pi͡atiznachnykh Chisel na dvukhznachnye. Umnozhenie li͡ubykh Chisel, Delenie i Prot͡sentirovanie s Tochnymi 4 i 6 Znakami* [Tables of products of five-figure numbers by two-figure numbers. Multiplication of any numbers, division and percentage-making, correct to 4 and 6 places]. *Posobie dli͡a Statistikov, Ekonomistov, Inzhenerov, Bukhgalterov i Proch.* [Aid for statisticians, economists, engineers, bookkeepers, etc.] Novocherkassk, Izdatelstvo Donskogo Politekhnicheskogo Instituta, 1930, 201 p. 25.7×35 cm.

69. J. R. Airey, "Bessel functions of nearly equal order and argument," *Phil. Mag.*, s. 7, v. 19, Feb. 1935, p. 230–235. 17×25.3 cm.

70. J. R. Airey, "The Bessel function derivatives $\frac{\partial}{\partial\nu}\cdot J_\nu(x)$ and $\frac{\partial^2}{\partial\nu^2}\cdot J_\nu(x)$," *Phil. Mag.*, s. 7, v. 19, Feb. 1935, p. 236–243. 17×25.3 cm.

71. Jiří Kaván, *Rozklad všetkých čisel celých od 2 do 256000 v Prvočinitele. Tabula Omnibus a 2 usque ad 256000 numeris integris omnes divisores primos praebens. Editio stereotypa.* (Publikácie štátneho astrofyzikálneho Observatoria, Stará Ďala, Ceskoslovensko.) Prague, Typis B. Stýblo, 1934, xi, 514 p. 29×30 cm. [This above title is taken from parts of two title pages.]

72. K. KARAS, "Tabellen für Besellsche Funktionen erster und zweiter Art mit den Parametern $\nu = \pm\frac{2}{3}, \pm\frac{1}{4}, \pm\frac{3}{4}$," *Z. f. angew. Math. u. Mechanik*, v. 16, Aug. 1936, p. 248–252. 20.2×28.6 cm.

73. H. TALLQUIST, "Sechsstellige Tafeln der 16 ersten Kugelfunktionen $P_n(x)$," Finska Vetenskaps-Societeten, Helsingfors, *Acta* n.s. A, v. 2, no. 4, 1937, 43 p. 22.3×28.4 cm.

74. F. TÖLKE, *Besselsche und Hankelsche Zylinderfunktionen nullter bis dritter Ordnung vom Argument* $r\sqrt{i}$, Stuttgart, Konrad Wittwer, 1936, 92 p. 17.6×25 cm.

J. R. Airey, Nos. **53, 54, 63, 69, 70**; E. S. Allen, No. **49**; Astronomicheskiĭ Institut, No. **45**; N. G. W. H. Beeger, No. **17**; L. Bendersky, No. **27**; J. Boccardi, No. **9**; J. Bouman and W. F. DeJong, No. **46**; E. W. Brown and D. Brouwer, No. **25**; R. S. Burington, No. **3**; V. R. Bursian, No. **59**; E. McC. Chandler, No. **39**; H. T. Davis, Nos. **24, 37**; L. E. Dickson, Nos. **16, 55**; A. Dinnik, No. **11**; F. J. Duarte, No. **8**; H. B. Dwight, No. **30**; F. E. Fowle, No. **31**; A. Gérardin, No. **13**; H. Gupta, No. **52**; W. L. Hart, No. **2**; P. Harzer, No. **28**; K. Hayashi, No. **12**; P. Herget, No. **4**; S. Hoppenot, No. **48**; E. L. Ince, Nos. **22, 41**; E. Jahnke and F. Emde, No. **32**; B. W. Jones, No. **57**; K. Karas, No. **72**; D. Katz, No. **34**; Jiří Kaván, No. **71**; M. Kraïtchik, No. **36**; M. Kraïtchik and S. Hoppenot, Nos. **14, 35, 40, 44**; A. M. Legendre, No. **51**; O. Lohse, No. **47**; H. J. Luckert, No. **26**; J. P. Möller and H. Q. Rasmusen, No. **64**; L. Ia. Neĭshuler, Nos. **67, 68**; Noordhoff, No. **1**; B. V. Numerov, Nos. **5, 6**; A. S. Percival, No. **29**; J. Peters, A. Lodge, and E. J. Ternouth, E. Gifford, No. **62**; J. Plassmann, No. **7**; P. Poulet, No. **43**; G. Prévost, No. **21**; H. Roussilhe and Brandicourt, No. **58**; J. B. Russell, No. **20**; S. Sakamoto, No. **66**; N. Samoilova-Iakhontova, No. 50; W. J. Seeley, No. **33**; J. Sherman, No. **18**; R. C. Shook, No. **56**; L. Silberstein, No. **61**; F. W. Sparks, No. **38**; H. Tallquist, Nos. **10, 73**; A. J. Thompson, Nos. **23, 42, 65**; F. Tölke, No. **74**; F. Triebel, No. **60**; A. Umansky, No. **19**; H. W. Weigel, No. **15**.

R. C. A.

75[D].—ROBERT ELMO BENSON (1903–), *Natural Trigonometric Functions Containing the Natural Sine, Cosine, Tangent and Cotangent to Seven Decimal Places for every Ten Seconds of Arc from* 0° *to* 90° *Semi-Quadrantally arranged.* Third edition, Los Angeles, California, Mac Printing Co , 133 East Third St., 1941. viii, 181 p. 15.2×23.2 cm. The first edition of this book appeared in 1927, and the second edition in 1929.

As calculating machines came into use, the need for seven-place tables of the natural values of the trigonometric functions stimulated a number of authors to prepare them. The first was by C. L. H. M. JURISCH, *Tables Containing Natural Sines and Cosines to Seven Decimal Figures of all Angles between 0° and 90° to every ten Seconds* . . . , Cape Town, 1884. 18×25.5 cm.; second ed. 1898; third, 1904; fourth (last) 1923. Then W. Jordan's work, prepared mainly for geodesists, *Opus Palatinum Sinus- und Cosinus-Tafeln von 10″ zu 10″*. Hannover, 1897. 16×24.7 cm.; second ed. 1913; third, 1923; sixth, 1936. In 1942 L. J. Comrie reported to me that *New Manual of Natural Trigonometrical Functions to Seven Places of Decimals of Sines and Cosines of Angles from* 0 *to* 10000 *Seconds*, published at New York by Wm. Chas. Mueller, 1907, is simply a pirated issue of the first edition of this Jordan work. And then the first table containing four functions, by H. BRANDENBURG, *Siebenstellige trigonometrische Tafel für Berechnungen mit der Rechenmaschine enthaltend die unmittelbaren, natürlichen Werte der vier Winkellinien—Verhältnisse, Sinus, Tangens, Cotangens und Cosinus* . . . *von* 10 *zu* 10 *Sekunden* . . . Leipzig, 1923. 20×27.8 cm. This edition contained, according to Mr. Comrie, over 400 errors. But the second edition, 1931, with prefaces in German, English, French, Spanish, and Japanese, "is practically free from error." And finally New Zealand, Department of Lands and Survey, *Natural Sines and Cosines for Every Ten Seconds of Arc to Seven Places of Decimals*, Wellington, 1927. 20.6×32.7 cm.

All four of these tables are directly or indirectly based on the two following extraordinary works:

I. o GEORG JOACHIM VON LAUCHEN, called Rhaeticus (1514–1576) and Valentin Otho, *Opus Palatinum de Triangulis*, [2 v.]. Neustadt, 1596. 21.6×35.8 cm. The second folio volume of 540 pages contains the complete ten-decimal trigonometric canon for every ten seconds of the quadrant, semiquadrantally arranged, with differences for all of the tabular results throughout. At the time

of his death Rhaeticus "left this canon all but complete; and the trigonometry was finished and the whole, edited by Valentin Otho under the title 'Opus Palatinum,' in honor of the Elector Palatine Frederick IV, who bore the expense of publication." At the end of the first volume is an excessively inaccurate canon (180 p.) of cosecants and cotangents to 7 places for every 10 seconds. In the ten-decimal canon August Gernerth found 598 errors, 130 of them being in the differences. These are all listed in *Z. f. d. österr. Gymnasien*, Heft VI, 1863, p. 414–425.

II. Bartholomäus Pitiscus (1561–1613), *Thesaurus Mathematicus sive Canon Sinuum ad radium* 1.00000.00000.00000 . . . Frankfurt, 1613. 24×35 cm. This work contains (a) natural sines for every ten seconds throughout the quadrant, to 15D, with first, second, and third differences; and (b) natural sines for every second, 0° to 1° and from 89° to 90°, to 15D, with first and second differences. Both of these tables were calculated by Rhaeticus. Pitiscus added, to 25D, the following: sin 30°, sin 15°, sin 5°, sin 1°, sin 30′, sin 10′, sin 5′, sin 1′, sin 30″, sin 10″, sin 5″, sin 1″; and sines for every 20″ for the first 35′ (beginning with 10′), to 22D, with differences. Such are the works on which the volumes listed above were based. Gernerth lists also, *l.c.*, p. 426–428, 110 errors which he found in this canon of sines.

The first edition (1927) of Mr. Benson's work was published before the second edition of the Brandenburg volume, but after the appearance of such publications as

M. H. Andoyer, *Nouvelles Tables Trigonométriques Fondamentales*, 3 v. Paris, 1915–1918. 24×31.3 cm. The complete trigonometric canon to 15D for every 10″ of the quadrant; and for [0°(9′)45°; 17D]. And

E. Gifford, *Natural Sines to Every Second of Arc and Eight Places of Decimals*, Manchester, 1914. 15.2×24.3 cm.; second edition, 1926. Also *Natural Tangents* . . . 1920 and 1927. Some of the errors of one of these volumes are dealt with in MTE **1**.

Mr. Benson does not mention the source of his tables, but L. J. Comrie has reported on the first edition (Royal Astr. So., *Mo. Notices*, v. 92, 1932, p. 340), "A thorough checking of the table revealed 150 errors, which, apart from a few copying or proof-reading errors, can be found in Rhaeticus, and are also in Brandenburg, who openly stated the source of his values [Rhaeticus]." The preface of the second edition (1929) contains the statement that "the few errors which were found existing in the first edition have been carefully corrected by the printer and it is now believed that these tables are absolutely correct." In the present edition this sentence starts "The few errors which were found existing in the first and second editions . . . ," but otherwise the two sentences and prefaces are identical. A rearrangement of pages of the first edition brought successive half-degree ranges of values of sines and cosines opposite the corresponding values for tangents and cotangents. In response to inquiries Mr. Benson wrote to me as follows in December, 1942:

"My table of *Natural Trigonometric Functions* was made from original calculations. The functions were calculated to 12 decimal places at various intervals, averaging about every 10 minutes. The intermediate functions were then derived by interpolation from these 12 decimal functions. Upon completion, the sines and cosines were compared with the *Opus Palatinum* of Jordan. The same procedure was followed in compiling the tangents, except that due to my anxiety to complete a very monotonous labor I succumbed to the temptation to take most of the intervening functions from Brandenburg [1923]. I discovered quite a number of errors most of which were obvious transpositions. These, of course, were corrected in my manuscript.

"Now as to the accuracy of my tables in their present form. In the first edition there were found and reported to me, seventeen errors. These were all corrected before the second edition was printed in 1929. When Mr. Ives compiled his table [first edition, 1931; see RMT **76**] he very carefully compared them with mine and found the 150 errors that you mentioned. These errors are all in the tangent table and are in every case an error of 1 in the seventh figure. Mr. Ives and I discussed this at some length and agreed that the value derived would not justify the expense of making so many corrections. Consequently these errors still exist in the present edition. I realize that to any mathematical purist this is unforgivable, but to a practising engineer it is of no consequence. To the best of my knowledge and belief there are no errors in the table of sines and cosines."

R. C. A.

76[D].—Howard Chapman Ives, *Natural Trigonometric Functions to Seven Decimal Places for Every Ten Seconds of Arc Together with Miscellaneous Tables*, Second Edition, New York, John Wiley & Sons, 1942, vi, 351 p. 17×24.9 cm.

The first edition of this book was published in 1931 and the present edition has been enlarged by 22 pages, with new tables, to 7D, for every 10″, for cotangents from 0° to 2° and for tangents 88° to 90°. These are cut down to 3 to 6 places in the following main table (semi quadrantal) of sines, cosines, tangents, and cotangents (p. 23–292), and indeed the cotangent is given to only 6D on to 5°42′30″. Each page is devoted to 10 minutes of the functions, with differences and proportional parts (not found in Benson's volume).

Then follow eleven other miscellaneous tables, namely: Length of arc to radius unity; Coefficient K for central angles of certain curves; Radii from arc definition; Radii from chord definition; Curves with even foot radii; Functions of a 1° curve; Corrections to tangent distances; Corrections to external distances; Trigonometric functions, formulas, and solution of triangles; Minutes in decimals of a degree; Units of length and of surface.

In the preface to the new edition it is stated that all errors found in the previous edition have been corrected for this edition. There is nothing in the prefaces of either edition which makes definite claim that the tables were compiled from original calculations. Indeed there is a passage which suggests rather the reverse; that, at least the table of tangents and cotangents are based on "fifteen place tables" computed by others. Of fifteen-place sexagesimal tables published before 1931 there are, in addition to the works of Rhaeticus and Andoyer (see RMT 75), the following:

H. BRIGGS, *Trigonometria Britannica* [the trigonometry by H. Gellibrand, but the tables calculated by Briggs and published after his death]. Gouda, 1633. 21.4×23.9 cm. Natural sines to 15 D, and tangents and secants to 10D, for every 36″, or .01°, of the quadrant. See RMT 79.

A. M. LEGENDRE, *Traité des Fonctions Elliptiques et des Intégrales Eulériennes, avec des Tables pour en faciliter le Calcul numérique*, v. 2, Paris, 1826, p. 252–255. 21×27 cm. Sines for every 15′ of the quadrant.

J. PETERS, "Einundzwanzigstellige Werte der Funktionen Sinus und Cosinus . . . " Preussische Akademie der Wissenschaften, *Abhandlungen, mathem. physik. Classe*, 1911, Anhang; also separately printed, 54 p. 22.7×26.4 cm. Sines and Cosines to 21 places of decimals for every 10′ of the quadrant and for every second of the first ten minutes with the first three differences.

C. E. VAN ORSTRAND and M. A. SHOULTES, "Values of sine θ and cosine θ to 33 places of decimals for various values of θ expressed in sexagesimal seconds," Washington Academy of Sciences, *Journal*, v. 12, p. 423–436, 1922. 17.3×25.8 cm. The range is 0″(1″)100″ (100″)1000″(1000″)45°.

The authors of printed sexagesimal tables of natural trigonometric functions, before 1931, to more than seven, but less than fifteen, places of decimals were G. J. Rhaeticus (1596); Pitiscus (1599, second ed. 1608, third ed. 1612); E. Gifford, Sines (first ed. 1914, second ed. 1926), Tangents (1920 and 1927). DeMorgan has told us that "Pitiscus will always be remarkable as the priest who wished that all his bretheren were mathematicians, to make them manageable and benevolent."

After the publication of the first edition of Ives's work, L. J. Comrie stated (Royal Ast. So., *Mo. Notices*, v. 92, 1932, p. 340–341) that the fifteen-place tables used by Ives, are "not those of Andoyer as one might imagine, but those of Pitiscus [Rhaeticus] as is shown by the 50 errors found by comparison with Andoyer. Most of the errors in Ives are also in Benson and the first edition of Brandenburg—all derived from the same source. From internal evidence it is clear that neither Benson nor Ives has printed from plates, thus introducing the grave risks associated with movable type." Since there were no early fifteen-place tables of tan and cot, possibly Mr. Comrie implies that for these functions the ten-decimal canon of Rhaeticus (1596) was used. On the other hand in response to my inquiry Mr. Ives made the following statement in a letter of 30 Nov. 1942: "My tables were not based on anything special. Almost all of the common books were used in checking, Andoyer, . . . , Benson, Trautwine, Brandenburg, Gifford, etc. For the most part Andoyer's 15 place tables were consulted in checking the last place . . . There were some 13 typewritten errors in my first edition. . . . As for the tables of Rhaeticus, while most of the later tables might be based on that, I never saw a copy of the book." Mr. Ives states also that his book was printed "from plates."

It may be noted that Mr. Ives published also

Seven Place Natural Trigonometrical Functions . . . , New York, Wiley, 1929, 10.3×17.4 cm. The six trigonometric functions, and versed sines, and external secants (secant-1), are given for every sexagesimal minute.

R. C. A.

77[D].—U. S. Coast and Geodetic Survey, *Special Publication*, no. 231, *Natural Sines and Cosines to Eight Decimal Places*, Washington, Government Printing Office, Washington, D. C., 1942, 541 p. 19×27.3 cm. Copies are sold by the Superintendent of Documents, Washington, D. C.

There is not one word of preface, text, or explanation in the volume, and the first page of the table is on the back of the titlepage. The table is for every second of the quadrant, and each page contains the results for five minutes of arc. Recalling recent tables covering exactly this same range one wonders why it should ever have been prepared and published. There was Emma Gifford's *Natural Sines to Every Second of Arc, and Eight Places of Decimals, computed . . . from Rheticus*, Manchester, 1914, iv, 544 p. 15.2×24.2 cm. Second edition, type reset, slightly smaller pages, 1926; and also the volume of Peters and Comrie (see RMT 78), *Achtstellige Tafel der trigonometrischen Funktionen für jede Sexagesimalsekunde des Quadranten*, Berlin, 1939. Being curious as to the possible relation of the new table to these or to other tables, in August 1942 I made application to the Survey for information, and received from its Acting Director, Mr. J. H. Hawley, the following facts: "The values in our *Special Publication* no. 231 were largely taken from the second edition of *Natural Sines to Every Second of Arc and Eight Places of Decimals* by E. Gifford. Numerous checks were made by comparison with the values given in *Nouvelles Tables Trigonométriques Fondamentales* by H. Andoyer [v. 1, Paris, 1915; 15 place table of sine and cosine for every ten seconds of arc]. Our tables were very carefully checked by differencing and by making horizontal and vertical summations according to patterns which we found workable. Our volume was not compared with the Peters table. This would undoubtedly be a desirable thing to do but we do not have the personnel available to undertake it at the present time. We believe that our table is reasonably free of errors and in much more convenient form to use than the Gifford table" [that is, semiquadrantal rather than the quadrantal display]. Before the volume was distributed the publisher had corrected the errors in sin 41°21′49″ and 54″. L. J. Comrie compared the entries of the Survey volume for five full degrees with the corresponding Peters entries. It was thus shown that for one ninth of all the entries in the volume there were only last-figure unit errors. Hence Mr. Comrie believes that the table may be regarded as reliable.

The manuscript for this work was prepared in 1941 as a Project of the Work Projects Administration in Philadelphia, under the sponsorship of the U. S. Coast and Geodetic Survey, L. G. Simmons, senior geodetic engineer, in charge. Nearly 1280 errors were found in Gifford's table and Mr. Hawley kindly placed these at our disposal for publication (see MTE 1).

R. C. A.

78[D].—JOHANN THEODOR PETERS (1869–), *Achtstellige Tafel der trigonometrischen Funktionen für jede Sexagesimalsekunde des Quadranten*, Berlin, Verlag des Reichsamts für Landesaufnahme, 1939, xii, 901 p. 20.9×29.7 cm.

In the year that this volume was published Mr. Peters was 70 years of age and was already an emeritus "Observator" and "Professor" (appointed 1901) of the Astronomisches Recheninstitut of the University of Berlin. In 1909 he started the publication of the 15 volumes of remarkable mathematical tables which have made his name famous. In recognition of this great contribution to science the Prussian Academy of Sciences awarded him the Leibnitz Medal. By 1911 he had published his 7-and 8-place tables of the logarithms of the trigonometric functions for every second of arc. At the meeting of the Astronomische Gesellschaft in Budapest in August 1930 Mr. Comrie, then director of the Nautical Almanac Office in Greenwich, discussed with Dr. Peters the calculation of twelve-figure values of the six trigonometric functions for each second of arc. By 1935 Mr. Comrie had completed the calculations for the cotangent and cosecant, and

the copy for 7-figure and 8-figure tables of the four principal functions. Mr. Peters made independent calculations of all six functions. It had been originally planned to print the calculated values to seven, eight, and ten figures in the three sets of three volumes each. The first volume of each set was to contain the sin and cos functions, the second the tan and cot functions, and the third the sec and csc functions. But of these 9 volumes planned a publisher (the German Government) was found for only two, in the volume before us, for sin, tan, cot, cos. On each page are the values for 3′. An introductory table (p. vi–xi) contains eight-figure values of w'' cot w for every $10''$ for $w=0°$ to $w=2°$; whence cot $w=(w''$ cot $w)/w''$ is readily found. So also for tan w near 90°.

The calculations of Peters were based on Andoyer's *Nouvelles Tables Trigonométriques Fondamentales*, v. 1–3, Paris, 1915–1918, in which the values of the six trigonometric functions are given for every ten seconds of the quadrant, divided into 90°. For the functions sin, cos, tan, sec, every fifth value, with twelve decimals, was extracted from these tables. The values were then checked by differences up to the fourth order, and 49 new values interpolated between every two consecutive values. (The method is described in detail in J. T. Peters, *Zehnstellige Logarithmentafel*, v. 1, Berlin, 1922, p. xi–xii.) The great care taken in proofreading included the checking by Mr. D. H. Sadler, the present director of the Nautical Almanac Office, of the accuracy of abbreviation of twelve-figure to eight-figure values.

For the four natural functions sin, tan, cot, and cos, this table undoubtedly supersedes all other existing eight-place tables, in point of view of accuracy, arrangement, and printing.

Comparatively few copies of this table had been sent out of Germany when the second world war had started. Since it was of value for various phases of the war effort, the British War Office made for its own use, but not for sale, even to British scientists, a facsimile edition of the Peters-Comrie work, with an English title page, foreword, and introduction. The title page is as follows:

Eight-figure Table of the Trigonometrical Functions for every Sexagesimal Second of the Quadrant by Professor Dr. J. Peters. *By order of the Reich Minister for the Interior, published by the Reich Survey Office.* Berlin, Publishers of the Reich Survey Office, 1939. 20×28 cm. Thus the paper pages (but not the tables) are somewhat smaller. The paper is not of as good quality as the original. On page 902 we find "Printed under the authority of His Majesty's Stationery Office," December, 1939. There was a second printing in 1940.

For certain information about the seven-place table for every second of arc, see the report of L. J. Comrie, president of Commission 4 (Ephemerides) of the International Astronomical Union, *Transactions*, v. 6, 1939, p. 359–361. Portraits of Mr. Peters and of Mr. Comrie, as well as brief biographical notes, may be seen in *Porträtgallerie der Astronomischen Gesellschaft*, Budapest, 1931.

R. C. A.

79[D].—J. T. Peters, *Seven-Place Values of Trigonometric Functions for every Thousandth of a Degree. Published and distributed in the Public Interest by authority of the Alien Property Custodian under License #1*. New York, D. Van Nostrand Co., 1942, [vi, 368] p. 17.7×23.7 cm.

This is a work for which copyright is "vested in the Alien Property Custodian, 1942, pursuant to law," and it is a reproduction of *Siebenstellige Werte der Trigonometrischen Funktionen von Tausendstel zu Tausendstel des Grades*, Berlin-Friedenau, 1918; reprinted, Leipzig, Teubner, 1930. The dimensions of the pages of this original volume are each 7–8 cm. larger, but the type pages of both volumes are identical in size. The Introduction of the new table is now in English as also are the headings of the tables on the last five pages, "Conversion of minutes and seconds to decimal parts of a degree," "Conversion of decimal parts of a degree to minutes and seconds," "Conversion of degrees to time," Conversion of time to degrees." The main tables give natural sines for every 0°.001 throughout the quadrant, followed by tangents at the same interval. Proportional parts are also given. This very inconvenient arrangement was dictated by the optical company for which they were prepared. There is also a one-page table of sin and tan [0°.00(0°.01)0°.58; 8 or 9D], and therefore of cos, cot [89°.42(0°.01)90°; 8 or 9D].

It is of special interest that the basis of the preparation of these tables was a work prepared more than three hundred years ago by Henry Briggs (1561–1631), professor of geometry at the University of Oxford from 1619. His tables were published posthumously at Gouda in 1633, at Vlacq's expense, in *Trigonometria Britannica*. They were of natural sines (to 15D) and tangents and secants (to 10D), also log sines (to 14D) and tangents (to 10D), at intervals of a hundredth of a degree from 0° to 90°, with interlined differences for all of the functions. These were all but completed by Briggs at the time of his death. They were published, with a work on trigonometry by Henry Gellibrand (1597–1637), professor of astronomy at Gresham College, London. In the work before us these tables are thus incorrectly referred to as Gellibrand's.

It is indeed extraordinary not only that Briggs should have conceived such tables as of high importance, but also that they should be of such remarkable accuracy. Peters found only two errors in all the sines and tangents—Briggs was correct where he listed a third error—but he overlooked other errors noted by Amelia de Lella (see MTE 2). Peters tested the accuracy of the Briggs table by differencing. Nine new values were inserted between each two of his values. "As in doing this no effort was made to obtain an accuracy of one-half of a unit in the seventh place, but simply not to overstep an accuracy of one unit in that position, which is sufficient accuracy for the needs of practical calculators, consideration of first differences only was satisfactory for the interpolations throughout the table of sines. The larger part of the table of tangents from 0° to 72° was dealt with in the same way. From 72° to 89°, second differences were also taken strictly into account in the interpolations." For the tangents from 89° to 90° Peters first calculated the logarithms of the tangents from 89° to 90° by the method explained in the introduction of J. Bauschinger and J. T. Peters, *Logarithmisch-Trigonometrische Tafeln mit acht Dezimalstellen*, Leipzig, 1910, and then found their antilogarithms. "The possible error does not amount to one unit of the seventh decimal place at any place in the tables; in the entire table of sines the greatest possible error is ± 0.65 units of the seventh decimal; in the table of tangents the error may rise to ± 0.85 in very rare cases." The Introduction includes a dozen illustrative examples of the use of the tables, primarily prepared for the machine computer.

The tables having been reproduced by some sort of photolithographed process, are naturally not as clear as in the original work, but such defects are of no great consequence. Since they are the only tables of their kind,[1] they will surely continue to serve a very useful purpose in varied types of research.

In conclusion the reader is reminded of Wilhelm Oswald Lohse (1845–1915), *Tafeln für numerisches Rechnen mit Maschinen*, Leipzig, 1909, vi, 123 p. 20.8×29.7 cm.; second newly rev. ed. by P. V. Neugebauer, 1935, vi, 113 p. 17×24.2 cm. See RMT 47. This volume contains the natural values of the six trigonometric functions for every 0°.01, with differences. Mr. Comrie has reported that this table is "especially convenient for orbital work." He has also warned that the similar table based on Briggs and compiled by Amelia De Lella, "should be avoided."

R. C. A.

80[D].—Leo Hudson and E. S. Mills, *Natural Trigonometric Functions. Tables. Sine, Cosine, Tangent, Cotangent, Secant and Cosecant to Eight Decimal Places with Second differences to ten Decimal Places Semi-quadrantally arranged. Instructions for use with the Monroe Adding Calculator.* Orange, N. J., Monroe Calculating Machine Co., 1941. 20+[47] p. 17.5×25.5 cm.

On pages up to 13 is material of elementary trigonometry, and then through page 20 are the "Instructions." The Tables occupy the remaining unnumbered pages. The title-page might easily mislead one who had not seen the book. Each page is devoted to one degree, and sines, cosines, tangents, and secants, are given for every minute, 0°–45°, to 8D; cosecants, cotangents, sines, cosines, are similarly presented for the range 45°–90°. Thus tangents and secants for this latter range, as well as cotangents and cosecants for the range 0°–45° cannot be read directly from the table. It is explained that these may be found by machine calculations of reciprocals. Beside each

[1] Gellibrand gave in his trigonometry (*l.c.* p. 43–44) sin x for $x = 0°.000(0°.625)90°$; 19D. Thus 144 values of the table under review are here carried to many more places of decimals.

of the columns tabulated is a column headed "Diff. per second"; e.g. between 10′ and 11′ we find 484.82 for sin and tan, and 1.48 for cos and sec; between 44°59′ and 45° the "diff" for sin and tan mount to 342.87 and 969.35 respectively. For the true differences these numbers are each to be multiplied by 10^{-8}. This is the interpretation of the phrase on the title-page, "with second differences to ten decimal places."

We are told that the sines were calculated from the formula

$$\sin x = x - x^3/3! + x^5/5! - x^7/7! + \cdots.$$

"The sine values were calculated until the ninth decimal place became stable and when it was 5 or greater the eighth decimal was increased by one; where it was less than 5, it was dropped. The Difference-per-Second values were calculated by dividing the difference between the minute values to nine decimal places by 60, carrying the calculation to ten decimal places, and equating the tenth decimal place to the nearest integer." The values of tan and sec were computed from $\tan x = \sin x/\cos x$; and $\sec x = 1/\cos x$, respectively.

Two errata are on a slip pasted in the book opposite the first page of the table. The volume is strongly bound, and clearly printed, but it is very expensive—about 9 cents a page. If a computer desires to use an 8-place table of the natural functions, it is obvious that the volume by Peters (see RMT **78**) would be decidedly preferable to the present volume. But if it turns out that one may depend upon the accuracy of this table, it may be well for the computer to know of this table when the Peters volume is not available. Comparison of the two volumes in a score of random angles showed constant agreement.

R. C. A.

81[D].—PROJECT FOR COMPUTATION OF MATHEMATICAL TABLES, *Tables of Sines and Cosines for Radian Argument*. Prepared by the Federal Works Agency, Work Projects Administration for the State of New York, conducted under the sponsorship of the National Bureau of Standards. New York, 1940, xxii, 275 p. 20.9×27.1 cm. Reproduced by a photo offset process. Sold by the U. S. Bureau of Standards, Washington, D. C.

Elsewhere in this issue is a reference to an account of the organization and achievements of the New York Mathematical Tables Project, of which Arnold N. Lowan is the technical director. Among its publications the volume under review is the third.

One of the main purposes of tables of sines and cosines with radian argument is to facilitate rapid calculation of transcendental functions from asymptotic expansions. Interpolation in calculating

$J_{1/2}(x) = (2/\pi x)^{1/2} \sin x$, and $J_{-1/2}(x) = (2/\pi x)^{1/2} \cos x$, *e.g.*, requires such tables.

It seems desirable that we should sometime survey tables of circular functions of angles other than those in sexagesimal units. Hence we shall here begin by listing tables with radian arguments, with their ranges, commencing with the one under review. This is a table of sines and cosines for the ranges [0.000(0.001)25; 8D], p. 1–250, [0(1)100; 8D], [0(10^{-6})0.01; 12D], [10^{-t}(10^{-t})9×10^{-t}; t=5, 4, 3, 2, 1; 15D]. There are also a table of $p(1-p)$, for p=0.000(0.001)0.500; and conversion tables of radians to degrees, and of degrees to radians.

L. J. Comrie recently suggested to A. N. Lowan the addition to this volume of two more pages (250A and 250B) printed so as to take the table to 25.2, i.e. just beyond 8π. "It is a great pity, when using a progressing argument to have to sacrifice a lot of the book by subtracting 6π."

The description of the self-checking feature of the iterated process of computation, and the double-differencing test of the final manuscript, which was then reproduced in facsimile, tend to inspire belief in the view of the authors "that this table is free from error."

The Project has also prepared tables of tan x, cot x for the range [0(0.001)2; 8D or 8S]; these are soon to be published by the Columbia University Press.

Other radian tables are as follows:

CARL BURRAU (1867–), *Tables of Cosine and Sine of Real and Imaginary Angles expressed in Radians (Circular and Hyperbolic Functions)*. Berlin, 1907. 16.5×23 cm. The volume has also

German and French title pages and prefaces. The table of sines and cosines [0.000(0.001)1.609; 6D] occupies p. 1–9.

GEORGE FERDINAND BECKER (1847–1919) and CHARLES EDWIN VAN ORSTRAND (1870–), *Smithsonian Mathematical Tables, Hyperbolic Functions*, Washington, Smithsonian Institution, 1909. 15×22.8 cm. The table of sin x, cos x, log sin x, log cos x, x= [0.0000(0.0001)0.100(.001)1.600; 5D] is on p. 173–223.

JOHN ROBINSON AIREY (1868–1937), Tables I–II, and ARTHUR THOMAS DOODSON (1890–), Table III, "Sines and cosines of angles in circular measure," Br. Ass. Adv. Sci., *Report*, 1916, p. 60–91. 13.7×21 cm. Table I for sines and cosines [0.000(0.001)1.600; 11D]. Table II for $\theta - \sin\theta$, $1-\cos\theta$, with first differences [0.00001(.00001)0.00100; 11D]. Table III for sines and cosines [0.0(0.1)10.0; 15D]. For second editions of Tables I and III see below.

C. E. VAN ORSTRAND, "Tables of the exponential function and of the circular sine and cosine to radian argument," National Academy of Sciences, *Memoirs*, v. 14, no. 5, 1921. 24.2×30.3 cm. Tables X–XIII, p. 47–78, give sin x and cos x for the following ranges: [0(1)100; 34D], [0.0(0.1) 10.0; 23D], [0.000(0.001)1.600; 23D], [1.10^{-t}, $2\cdot10^{-t}$, $\cdots$, $9\cdot10^{-t}$, where t=4, 5, $\cdots$, 10; 25D]. In Table XIV sin 0.1 and cos 0.1 are each given to 120D, and sin 1.0 and cos 1.0 to 105D.

J. R. AIREY, Br. Ass. Adv. Sci., *Report* (a) *1923*, p. 287–289, sin x and cos x [0(1)100; 15D]; (b) *1924*, p. 276–278, sin x and cos x [10.0(0.1)20.0(0.5)50.0; 15D]; (c) *1928*, p. 305–307, sin x and cos x [20.0(0.2)40; 15D]. The calculations of table (a) were based on the values of sin 1^r and cos 1^r, each to 105D, given by C. A. Bretschneider in *Archiv d. Math. u. Phys.*, v. 3, 1843, p. 29. The values to 52D, given in the Anhang, p. 60, of Peters, *Zehnstellige Logarithmentafel*, v. 1, 1922, are in agreement with these. The values of sin 1^r, cos 1^r to 25D, given by Christoph Gudermann, *Jn. f. d. reine u. angew. Math.*, v. 6, 1830, p. 11, are each a unit too large in the 25th place, while the value of cos 1^r in its 13th place should be "1," not "0." But H. S. Uhler has calculated sin 1^r, cos 1^r to 477D, Conn. Acad. Arts and Sciences, *Trans.*, v. 32, 1937, p. 433–434. On the latter page is also given sin 10^r, cos 10^r, cos 20^r to 212D; sin 100^r, cos 100^r, sin 200^r, cos 200^r to 72D. From Mr. Uhler's computations it may be checked that the corresponding results of Bretschneider, Airey, and Van Orstrand are wholly correct.

KEIICHI HAYASHI (1879–), *Sieben- und mehrstellige Tafeln der Kreis- und Hyperbelfunktionen und deren Produkte sowie der Gammafunktion* . . . , Berlin, Springer, 1926. 21×27.3 cm. There are tables of sines and cosines (p. 2–204) [0.00001(.00001)0.00100; 20D], [0.010(.0001)0.0999; 10D], [0.100(.001)0.999; 12D], [1.000(.001)2.999; 12D], [3.00(.01)9.99; 18D], [10.0(.1)20(1)50; 15D]. L. J. Comrie has told us that in this volume are more than 1500 errors (other than last figure errors).

K. HAYASHI, *Fünfstellige Funktionentafeln, Kreis-, zyklometrische, . . . Funktionen, . . .*, Berlin, Springer, 1930, 16.5×24.7 cm. On p. 2–40, 44–47, 162–164, are sin x, cos x, tan x, for x= [0(0.01)10.00; 5D]; tan x, sec x, x= [1.560(0.001)1.590; 5D]; sin x, cos x for x= [0(0.0001) 0.0100; and 10(1)40; 10D]; sin $\frac{x\pi}{2}$, cos $\frac{x\pi}{2}$ for x= [0.001 (.001)0.500; 5D]. This volume was based on the previous one. Hayashi is professor of engineering at the Kyushu Imperial University.

BR. ASS. ADV. SCI., Committee for the Calculation of Mathematical Tables, *Mathematical Tables*, v. 1, London, Br. Ass., 1931. 21.5×27.9 cm. On p. 3–7 sin x and cos x are given for x = [0.0(0.1)50.0; 15D] by A. T. Doodson, J. R. Airey, L. J. Comrie. This is followed (p. 8–23, by the Airey table of sin x and cos x for x= [0.000(0.001)1.600; 11D], first printed with the Doodson table in 1916, as we noted above; similarly for other Airey tables above (1923–1924).

It will thus be seen that the values of sin x and cos x in the New York volume for the range from 0 to 1.600 are contained in tables of Van Orstrand, and of the Br. Ass. Adv. Sci. volume. The same may be said of values for key arguments at intervals of 0.1, for the rest of the range.

Among other tables, useful for certain kinds of work, the following may be mentioned:

FREDERICK EUGENE FOWLE (1869–), *Smithsonian Physical Tables*. Eighth rev. ed., (*Smithsonian Miscellaneous Collections*, v. 88), Washington, D. C., 1933. 15×23 cm. Table 15, p. 37–40, Sin, Cos, Tan, Cot, and their logs. [0.00(.01)1.60; 5D].

N. H. KOLKMEIJER, "Trigonometric and exponential functions," *International Tables for the Determination of Crystal Structures*, v. 2, *Mathematical and Physical Tables*, Berlin, Gebrüder Borntraeger, 1935. 18.5×27.7 cm. On p. 546–550 is a table of sin $2\pi x$ and cos $2\pi x$ for $x=$ [0.000(0.001) 1.000; 4D]. But this table was extended by M. J. Buerger, *Numerical Structure Factor Tables* (Geological So. Amer., *Special Papers* no. 33) New York, 1941, 15.4×23.9 cm. Table 1, p. 12–111, is a double entry table for the values of cos $2\pi hx$, sin $2\pi hx$ for $x=$ [0.000(0.001)0.999; 3D], and for $h=1(1)30$. This is one of five tables for use in computation of the x-ray diffraction intensities connected with a given crystal structure. The computation involves the products of two or more sines or cosines of this form. If $y=(1+\cos^2 2x)/\sin 2\theta$, the last four tables are of y, $y^{1/2}$, y^{-1}, $y^{-1/2}$, to 3D, with argument sin $\theta=0.000(0.001)0.999$.

ULFILAS MEYER (1885–) and ADALBERT DECKERT, *Tafeln der Hyperbelfunktionen Formeln*, [Berlin, 1924]. 16.8×24.1 cm. On p. 22–30, 50–58, sin x, cos x, tan x, log sin x, log cos x, log tan x, are given for the range [0.000(0.001)1.569; 5D or S]. The scope of the volume was primarily determined by the needs of German telegraph engineers.

LOUIS MELVILLE MILNE-THOMSON (1891–) and L. J. COMRIE, *Standard Four-figure Mathematical Tables, including many New Tables, Trigonometrical Functions for Radians, . . . , Edition A, with positive Characteristics in the logarithms*, London, Macmillan, 1931. 19.2×26.5 cm. In Edition B the logarithms of numbers less than unity are printed with negative characteristics. Table IX (p. 96–131): (a) csc x, cot x, log sin x, log tan x, with differences, for the range [0.0000(0.0001) 0.0400; 4S]. (b) sin x, csc x, tan x, cot x, sec x, cos x, log sin x, log tan x, log cot x, log cos x, with differences, for the range [0.000(0.001) 1.570; 4S]. (c) sin x, cos x, log sin x, log cos x, for the range [0.0(0.1)7.9; 4D].

HENRI CHRÉTIEN, *Nouvelles Tables des Sinus Naturels Spécialement adaptées au Calcul des Combinaisons Optiques donnant les sinus, ou des arcs, directement, sans interpolation avec six ou cinq décimales et avec une décimale supplémentaire par interpolation ordinaire, suivies de tables de Tangentes Naturelles et de Conversion des Angles.* Paris, *Revue d'Optique Théorique et Instrumentale*, 1932. 16.5×26.5 cm. Table I (p. 9–14) is of $\Delta=x-\sin x$, for each $\Delta=0.000001(0.000001)0.001000$, x and sin x being given to 6D, so that x ranges from $0.^r014423$ to $0.^r181842$, i.e. to 10°25′. Table II (p. 15–27), $\Delta=0.00001(0.00001)0.02400$, to 5D, x ranging from $0.^r03107$ to $0.^r52661$, i.e. to 30°10′. Compare the table by Airey given above. Table III (p. 29–34) sin x, $x=$ [0.500(0.001)1.600; 5D], 28°39′ to 91°41′. Table IV (p. 35–39) tan x, $x=$ [0.000(0.001)1,000; 5D], to 57°18′. Tables V–VII (p. 40–42) are for converting radians, grades, and degrees, into one another. Chrétien was professor at the Institute of Optics of the Sorbonne. The advantages, in machine calculation, of the use of this table in comparison with others published before the volume under review, are set forth by L. J. Comrie, "The use of calculating machines in ray tracing," Physical So., *Proc.*, v. 52, 1940, p. 246–249; discussions, p. 250–252.

J. R. AIREY, "The circular and hyperbolic functions, argument $x/\sqrt{2}$" *Philosophical Mag.* s. 7, v. 20, 1935, p. 721–726. Tables of sin $(x/\sqrt{2})$ and cos $(x/\sqrt{2})$ for $x=$ [0.0(0.1)20.0; 12D]; sin $x/\sqrt{2}$ and cox $x/\sqrt{2}$ appear in the asymptotic expansion of $J_0(z)$ in the calculation of ber x and bei x, the argument z of the Bessel function being taken along the "semi-imaginary" axis $z=xe^{-i\pi/4}$. In preparing the tables some of the calculations were based on sin $1/\sqrt{2}$ and cos $1/\sqrt{2}$ each given to 20D, and sin $0.1/\sqrt{2}$ and cos $0.1/\sqrt{2}$ each to 18D. See the Note on p. 31 of this issue.

R. C. A.

82[A, B].—*Barlow's Tables of Squares, Cubes, Square Roots, Cube Roots, and Reciprocals, of all Integer Numbers up to 12,500, edited by* Leslie John Comrie (1893–). Fourth edition, London, E. & F. N. Spoon, Ltd., and Brooklyn, N. Y., Chemical Publishing Co., Inc., 234 King St., 1941. xii, 258 p. 14×21.7 cm.

In the early part of the nineteenth century Peter Barlow (1776–1862), professor of mathematics at the Royal Military Academy, Woolwich (1806–1847), and fellow of the Royal Society (1823), was one of the leading mathematicians of England. He published a volume entitled *An Elementary Investigation of the Theory of Numbers* (London, 1811), and his *A New Mathematical and Philosophical Dictionary* (London, 1814) is still a valuable source of information. His contribu-

tions to the magnetic theory, optics, and strength of materials were also notable. For other details about his numerous papers and books see *Royal Soc. Catalogue of Scientific Papers*, and R. Ast. So., *Mo. Notices*, v. 23, 1862, p. 127–128.

A second volume which he published in 1814 was

New Mathematical Tables, containing the Factors, Squares, Cubes, Square Roots, Cube Roots, Reciprocals, and Hyperbolic Logarithms, of all numbers from 1 to 10000; Tables of Powers and Prime Numbers; an extensive Table of Formulae, or General Synopsis of the most important particulars relating to the Doctrines of Equations, series, fluxions, fluents, etc. lxiv, 336 p., [**A, B, C, D, E, F, H, M**]. There are in the volume, X Tables, and a long introduction. Errata in the fourth and higher powers of integers, in Table III, are given by A. J. C. Cunningham in *Messenger Math.*, v. 35, 1905, p. 18–19. In 1840, on the suggestion of the Society for the Diffusion of Useful Knowledge, Augustus De Morgan (1806–1871) brought out a new edition of Table I, with the "factors" omitted, and with the following title, *Barlow's Tables of Squares, Cubes, Square Roots, Cube Roots, Reciprocals, of all Integer Numbers up to 10,000.* In preparing this edition De Morgan secured the services of Mr. Farley, of the Nautical Almanac Office, who discovered 14 errors in the squares, 33 in the cubes, 14 in the square roots, 7 in the cube roots, 22 in the reciprocals. These errors are listed in the stereotype reprints at least as late as 1866; they do not appear in the printing of 1897, for example.

The third edition of Table I, brought out by L. J. Comrie in 1930, was a great advance on De Morgan's edition. The page was enlarged to that of the original Barlow, and an additional column was introduced, showing the square root of (a) the reciprocal of a number up to 1000, and then (b) 10 times the number in the argument column, in other words giving the square root of every 10th number between 10,000 and 100,000. Differences are also introduced in the four columns after $n=1000$. In the preface Mr. Comrie gives most satisfying information as to his exceedingly thorough checking of various details of the De Morgan edition; there were no errors in the squares and cubes, but many errors in the other columns. He exhibits the extent to which portions of a number of other tables may be trusted. In this edition Mr. Comrie gives also the fourth to the tenth powers of all numbers from 1 to 100 (and n^4 to 1000), the 11th–20th powers of the integers 1–10, and two pages of binomial coefficients, constants, etc.

The Comrie edition of this work has never been allowed to go out of print, and there has al, ways been an American agent for the publication. In spite of this, in 1935 G. E. Stechert and Co.- of New York, brought out a pirated facsimile (14.6×23 cm.) of this third edition; it was made in Shanghai, China. Two changes occur on the title page, "New impression" is added under "Third Edition" and Stechert's name has been substituted for that of the American agent of 1930 and later. The page is somewhat larger but the printing is appreciably inferior to that of the English edition. The Russians perpetrated a similar piracy of the third edition in 1933.

In the present real fourth edition 50 pages have been added to the third edition, through extending the table from 10,000 to 12,500, "particularly in order to avoid discontinuities when working with numbers just below and just above unity." This edition is one of the indispensable tables for the computer to have at hand. The square roots and cube roots are to seven places, and the reciprocals to seven significant figures, i.e. nine places to 1000, and above this, ten. See MTE 4.

We understand that Mr. Comrie has in ms. a natural extension to this volume, namely: a table of $n^{-1/2}$, $(10n)^{-1/2}$ for $n=1000(1)12500$. We hope that its publication may not be long delayed.

R. C. A.

83[N].—*Financial Compound Interest and Annuity Tables computed by* Financial Publishing Company *under editorial supervision of* Charles H. Gushee. Boston, Financial Publishing Co.; London, Geo. Routledge & Sons, Ltd., 1942, 884 p. 13×23.3 cm.

This volume contains six principal tables, p. 2–733, arranged side by side, three on each of two facing pages. These tables give, for different rates and periods, the amount of 1, that is $s=(1+i)^n$; the amount of 1 per period, $s_{\overline{n}|}=[(1+i)^n-1]/i$; periodic deposit that will grow to 1 at a future

date, $1/s_{\overline{n}|}$; present worth of 1, $v^n=1/s$; present worth of 1 per period, $a_{\overline{n}|}=(1-v^n)/i$; periodic payments necessary to pay off a loan of 1, $1/a_{\overline{n}|}=i/(1-v^n)=i+1/s_{\overline{n}|}$.

The rates are 1% (1/4%) 7% (1/2%) 10%. These nominal annual rates are shown for annual, semiannual, quarterly or monthly compounding. There are 360 periods for monthly compounding, or 30 years; 240 periods for quarterly compounding; 240 periods for semiannual compounding; 120 periods for annual compounding. The results in the tables are to ten places of decimals, which means that 16 significant figures are given in some places.

Pages 735–779 are devoted to Auxiliary Tables for fractions $(1/p)$ and multiples (n) of a unit period. For $p=365, 360, 180, 52, 26, 13, 12, 6, 5, 4, 3, 2, 1$, $n=2, 3, 4, 6, 12, 13, 26, 52, 180, 360, 365$, we are given, mostly, to ten places of decimals, $s=(1+i)^{1/p}=1+i'$; $s_{\overline{p}|i'}=i/[(1+i)^{1/p}-1]=[(1+i')^p-1]/i'$; $1/s_{\overline{p}|i'}$, i' indicating the interest for a fraction of the period. For multiples of the unit period $s=(1+i)^n$; $s_{\overline{n}|i}=[(1+i)^n-1]/i$. The percentages tabulated are, per period, as follows: 1/12% (1/48%) 7/12% (1/24%) 7/8% (1/16%) 1 3/4% (1/8%) 3 1/2% (1/4%) 7% (1/2%) 10%.

A wide range of examples (78) illustrating the application of the tables is clearly set forth on pages 781–869. They include those for which adjustments or extensions of the tables are necessary to fit the problems. Interpolation within the tables (including example 70), pages 842–861, with tables of interpolation factors, would certainly guide the non-mathematical specialist in the right direction. Probably the same remark may be made about the section on "Non-financial applications," where the compound interest law for continuous compounding comes up, even though we do find there "Instantaneous compounding means that we add an infinitely tiny amount of interest an infinitely large number of times."

The authors are probably correct in stating that "The main virtue of this edition is that it shows these tables for a greater number of rates, and for a greater number of periods than have ever before been published in a single book." The whole arrangement seems eminently practical and convenient. Of course certain things not given in this volume could be found in other tables; for example, in P. A. Violeine, *Nouvelles Tables pour les Calculs d'Intérêts Composés d'Annuités et d'Amortissement*, neuvième edition, entièrement refondue par A. Arnaudeau, *nouveau tirage avec un supplément contenant les tables de* 8% *à* 15%, Paris, Gauthier Villars, 1924, 21.5×27 cm. We here find tables, to ten places of decimals, for s, $s_{\overline{n}|}$, and $1/a_{\overline{n}|}$, for $10\frac{1}{4}\%$ $(\frac{1}{4}\%)$ 15%, for $n=1, \cdots, 100$.

R. C. A.

84[K, O, P, S].—Edward Charles Dixon Molina (1877–), *Poisson's Exponential Binomial Limit. Table I—Individual Terms, Table II—Cumulated Terms.* New York, D. Van Nostrand Co., 1942. viii, 46, ii, 47 p. 21×27.9 cm.

The "Limit" to which reference is here made occurs in *Recherches sur la Probabilité des Jugements en matière criminelle et en matière civile, précédées des règles générales du Calcul des Probabilités* by Siméon Denis Poisson (1781–1840), Paris, 1837, p. 206. Jacques (James) Bernoulli (1654–1705) in his *Ars Conjectandi*, Basle, 1713, p. 38–42, states, in effect, that if the probability of an event is p, the probability of exactly x successes in n independent trials is

$$P = C_x^n p^x (1-p)^{n-x},$$

the $x+1$st term in the expansion of the binomial $(\overline{1-p}+p)^n$. In $P(c, n, p)=\sum_{x=c}^{n} C_x^n p^x(1-p)^{n-x}$, which is the probability of an event happening at least c times in n trials, we have a series of frequencies, in which Poisson allowed $n\to\infty$, and $p\to 0$, while $np=a$ is kept constant, and found as "limit"

$$P(c, a) = \sum_{x=c}^{\infty}\left(\frac{a^x e^{-a}}{x!}\right).$$

Mr. Molina's Table I is of $(a^x e^{-a}/x!)$, for $a=0.001(0.001)0.01(0.01)0.3(0.1)15(1)100$; $x=0(1)150$; to 6D. For $a=.001$, the only values of x are 0, 1, 2; but for $a=100$, $x=56(1)150$. Table II of $P(c, a)$ also to 6D is for the same range of a, and $c=0(1)153$. The volume was lithoprinted by Edwards

Brothers, Inc. and leaves a great deal to be desired. On very many pages there are numbers which are much blacker, and less distinct, than the other numbers, and in some places one cannot be sure what numbers are intended; for example, Table I, $a=14.4$, $x=14$; $a=19$, $x=21$; $a=81$, $x=68$. There are a number of cases of only parts of numbers being printed.

A table of $(a^x e^{-a}/x!)$, to 4D, was given by the Russian statistician Ladislaus von Bortkiewicz (1868–1931), in his *Das Gesetz der kleinen Zahlen*, Leipzig, 1898, p. 49–52, 16×23.7 cm., $a=0.1(0.1)10.0$; $x=0(1)24$. The fourth figure is inaccurate in many instances; also there are objections to the author's use of the term "law of small numbers," as applied to $P(c, a)$. H. E. Soper gave a six-place table of $(a^x e^{-a}/x!)$ in *Biometrika*, v. 10, 1914, p. 27–35, 19×27 cm., for $a=0.1(0.1)15.0$; $x=0(1)37$. This was also printed as Table LI in *Tables for Statisticians and Biometricians*, ed. K. Pearson, Cambridge, University Press, 1914; second ed. 1924.

Tables for $P(c, a)=.0001$, .001, and .01 respectively, with a running from .0001 to 928, appeared in Mr. Molina's article "Computation formula for the probability of an event happening at least c times in n trials," *Amer. Math. Mo.*, v. 20, 1913, p. 193. A similar table, with much more extensive values assigned to $P(c, a)$ was given by G. A. Campbell in *Bell System Technical Jl.*, v. 2, 1923, p. 108–110; also in *The Collected Papers of George Ashley Campbell*, New York, 1937.

Mr. Molina has here published independently prepared tables of great importance in many fields. Only small parts of Tables I and II have appeared in print earlier. He tells us how "The work of computing and checking which these tables represent is that of many individual members of the Bell System over the course of almost forty years." We are informed that these tables have been found especially useful in quality engineering work in finding the most advantageous plan for a given set of conditions. The table $P(c, a)$ is in a form directly usable for solving problems of "single, double, and multiple sampling which require the determination of compound probabilities relating to the occurrence of c or more, or c or less defective units in the first, second, etc., samples of stated sizes."

In H. Levy, *Elements of Probability*, Oxford, Clarendon Press, 1936, "The telephone problem," p. 144–145, it is remarked that the telephone service in operation presents an enormous number of practical problems in probability. These are, however, necessarily so technical that a simple case only, involving $P(c, a)$, is given in illustration. Extensive and interesting discussion of "the Poisson Law" and its applications, is to be found in T. C. Fry's *Probability and its Engineering Uses*, London, Macmillan, 1928; on p. 458–463 are tables for $a^x e^{-a}/x!$ for $a=0.1(0.1)1(1)20$, $x=0(1)44$; and for $P(c, a)$, a as before, and $x=0(1)45$.

R. C. A.

85[L, M].—J. C. Jaeger and Martha Jaeger, "A short table of $\int_0^\infty \frac{e^{-xu^2}}{J_0^2(u)+Y_0^2(u)} \frac{du}{u}$," Royal So. Edinburgh, *Proc.*, s.A, v. 61, no. 19, 1942, p. 229–230. 17.5×25.4 cm.

This table, from the University of Tasmania, follows a paper, no. 18, by Mr. Jaeger, "Heat flow in the region bounded internally by a circular cylinder" (p. 223–228). The object of the paper was "to give some numerical results for the cooling of the region bounded internally by a circular cylinder, with constant initial temperature, and various boundary conditions at the surface. Problems of this nature are of importance in connection with the cooling of mines, and in various physical questions." The discussion leads to integrals of the type

$$I(p, q; x) = \int_0^\infty \frac{e^{-xu^2}}{[puJ_1(u)+qJ_0(u)]^2+[puY_1(u)+qY_0(u)]^2} \frac{du}{u}.$$

When x is large, an approximation given in the paper readily yields numerical results. But when x is small we are led to $I(0,1; x)$ which is tabulated for x, [0.00(0.01)10(0.1)100(1)1000; 3D]. In all cases the values were "calculated to at least one more place than shown so it is hoped that the final figures are correct."

R. C. A.

86[C].—HORACE SCUDDER UHLER (1872–), *Original Tables to 137 Decimal Places of Natural Logarithms for Factors of the Form* $1 \pm n \cdot 10^{-p}$. *Enhanced by Auxiliary Tables of Logarithms of Small Integers.* New Haven, Conn. The Author, Sloane Physics Laboratory, 1942. [120] p. 21.3×27.7 cm. Published in an edition of 800 copies.

These tables will be of very little interest to the "practical computer." Occasionally one sees statements about the maximum accuracy necessary in tabulating logarithms for practical purposes. Although most of these statements have exceptions, it is absurd to expect any practical use for a table to 137 decimal places. It would be more correct to say that these tables are of practical value only in solving impractical problems. It is for just such a problem that the author has prepared these tables. He has made them available to the public in the hope that they may be used effectively on other problems of the same kind.

These tables give the natural logarithms of numbers of the forms $1+n \cdot 10^{-p}$ and $1-n \cdot 10^{-q}$ where n, p, and q are positive integers not exceeding 9, 21, and 69 respectively, together with the logarithms of certain small integers (2–11, 13, 17, 19, 23, 29, 31, 47, 53, 71, 97, 101, 103, 107, 109, 113) and of 10^k for $k=20, 30, 40, \cdots, 90$, and 110, all to 137 or more decimal places.

At first one might think that in publishing natural instead of common logarithms, the author has failed to recognize the fact that numbers are usually written to the base 10. On the contrary, the author is exploiting the base 10 to the utmost in choosing the natural system of logarithms. In fact, the natural logarithm of a number like $1+3 \cdot 10^{-8}$ is equal to

$$0.00000\ 00299\ 99999\ 55000\ 00089\ 999 \cdots$$

whereas its common logarithm is

$$0.00000\ 00130\ 28834\ 26166\ 47418 \cdots,$$

an unpredictable sequence of digits. In short since

$$\ln (1 \pm n \cdot 10^{-p}) = \pm n \cdot 10^{-p} - \tfrac{1}{2}n^2 \cdot 10^{-2p} \pm \tfrac{1}{3}n^3 \cdot 10^{-3p} + \cdots,$$

the first six terms of this series are rational numbers whose decimal expressions either terminate quickly or are periodic of period one so that the digits of this logarithm follow simple general patterns (given in Tables 5 and 6) until the term $\frac{1}{7}n^7 \cdot 10^{-7}$ is reached. This fact renders it unnecessary to give tables of natural logarithms of $1 \pm n \cdot 10^{-p}$ for p slightly more than $N/7$ where N is the number of decimal places desired. This is why p, as mentioned above, does not exceed 21. The last two-thirds of Table 2, giving $\ln(1-n \cdot 10^{-q})$ for $21 \leq q \leq 69$, is, in the opinion of the reviewer, largely a waste of printing. It is interesting to note that the decimal system, for once, is superior in this respect to the duodecimal system, since in the latter system the earlier term $\frac{1}{5}n^5 \cdot 10^{-5}$ gives trouble.

The general method of using a table of this sort to compute natural logarithms and exponentials is the same as in the familiar tables for common logarithms and anti-logarithms. (The only table of common logarithms comparable with these under review is an obscure table of Parkhurst[1] to 100 decimals.) In explaining the use of these tables the author chooses problems having about 15 decimals. The reader is left to discover for himself the technique of dealing with problems having over 100 decimal places.

The elaborate mechanical, semi-mechanical, and photographic methods used in calculating and checking the tables make it quite unlikely that a single error exists in the thousands of digits printed.

D. H. L.

1. Henry Martyn Parkhurst (1825–1908), *Astronomical Tables comprising Logarithms from 3 to 100 Decimal Places and other Useful Tables*, New York, 1868; other editions 1869, 1871, 1873, 1875, 1876, 1881, and 1889 (final). 11.3×15.1 cm. The volume was set up and electrotyped by the author himself, a "law stenographer." Some of the later editions have more than twice as many pages as the first edition. Tables II, III, IX contain, with 19 exceptions, log N to 102D, $N=1(1)$ 109. Memoirs by Parkhurst may be found in Harvard College, Astronomical Observatory, *Annals*, v. 18 (1890), 29 (1893). *The Catalogue of the Astor Library, Authors and Books L-R*, Cambridge, Mass., 1887, lists an "incomplete" copy of the first section of Parkhurst's *Tables*, as

87[A, C, P].—RAY M. PAGE, *14000 Gear Ratios. Tabulated Ratios presented in Common Fractional and Decimal Forms and in Differently arranged Sections to facilitate the Solution of all Classes of Gear-Ratio Problems.* New York, The Industrial Press, 148 Lafayette St.; and Machinery Publishing Co., 17 Marine Parade, Brighton, England, 1942. iv, 404 p. 21.5×27.7 cm.

There are four tables in this volume, p. 23–403. In Table I are the decimal equivalents of rational fractions arranged in 119 tables, each table being complete on one page with the denominator of the fraction serving as an index. These denominators range from 2 to 120 inclusive. The decimal equivalents are given to eleven places when the ratio is less than unity, and to nine places when greater. Such gear ratios are of importance in Machine Shops and Mechanical Engineering, where the gears generally used do not have more than 120 teeth. The tables here include any ratio from 1/120 to 120/2 resulting in 14,280 ratios which decimally start at 0.008333 . . . and continue to 60.000 . . . . As an illustrative example of the use of such ratios (*Machinery's Handbook*, eleventh ed., New York and London, 1941, p. 671) it may be required that the speeds of the driving and driven gears are to be as near as possible to 1149 and 473 revolutions per minute. It may be stipulated, however, that the number of teeth in the larger gear must not exceed 60. Dividing 473 by 1149 we find 0.41166 . . . . By referring to the tables, the nearest fractional value to this ratio with a denominator less than 60, is found to be 7/17. "Thus, the nearest number of teeth in the gears can be 14 and 34, or 21 and 51. This will give speeds of 1149 and 473.118 revolutions per minute, which introduces a very small error. In the absence of such tables, the method of obtaining the approximate fraction 7/17 would be very cumbersome." In the first 22 pages of the work under review are many examples of gear ratio and speed problems.

The decimal equivalents of the ratios in Table I are in Table II (p. 143–258) arranged, in somewhat abbreviated form, in order of magnitude, and each is followed by its logarithm, to 7D, and by equivalent ratios, sometimes very numerous. For example, .66667 . . . is followed by 40 ratios from $\frac{2}{3}$ to 80/120, all equivalent to $\frac{2}{3}$.

In Table III (p. 259–364) for a total number of teeth, the corresponding gear pairs and decimal equivalents are given for every integer from 25 to 239; under 25, for example, are 12/13 = .923077 and 13/12 = 1.083333. Table IV (p. 365–403) is a gear factor table and consists of all numbers from 20 to 14400 that are the product of two factors neither of which exceeds 120. This table is used in finding gear combinations equivalent to a given numerator and denominator. For example, if a ratio = 697/1081, the table shows that (17×41)/(23×47) is an equivalent expressed in practical gear sizes.

Mr. Page's collection of gear-ratio tables is incomparably more extensive than anything previously published. The first person to conceive of such tables in connection with gear ratios was the French horologist, Achille Brocot, and the first publication in this connection was a paper presented by Brocot at a meeting of the Société des Horlogers at Paris, in June, 1860, "Calcul des rouages par approximation, nouvelle méthode," *Revue Chronométrique, Journal des Horlogers, Scientifique et Pratique*, v. 3, p. 186–194. He here refers to his work with tables which he had ready for publication. Their titles are transcribed from the *Catalogue Général des Livres Imprimés de la Bibliothèque Nationale:*

o*Calcul des Rouages par Approximation, Nouvelle Méthode*, Paris, l'auteur, 1862. Gr. in 8°, 97 p.
o*Table de Conversion en Décimale des Fractions Ordinaires, à l'usage du Calcul des Rouages par Approximation. Nouvelle Méthode.* Paris, P. Dupont, 1862. In 8°, 51 p.

No copies of these works are to be found in any of the principal libraries of America. But it would seem as if the gist of Brocot's work may have been contained in the following publication:

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well as other publications (1854–77) on spelling reform, stenography, etc. He was the chief reporter of the U. S. Senate debates 1848–54, and one of the reporters of the *Debates* of the Maryland Constitutional Convention of 1864. The 1869 edition of the *Tables* is in Library of Congress and the 1868 and 1873 editions in the New York Public Library (Astor Lib.). There is a sketch of Parkhurst in *Pop. Astr.*, v. 16, 1908, p. 231–239, and portrait plate.—EDITOR.

Berechnung der Räderübersetzungen. Herausgegeben von dem Verein „Hütte." Bearbeitet nach Calcul des Rouages par Approximation, Nouvelle Méthode par Achille Brocot, Horloger. Second ed., Berlin, Ernst & Korn, 1879. 68 p. 11.4×17.8 cm.

This little volume contains two tables, of which the first, occupying p. 15–54, gives the decimal equivalent to eleven places of decimals of the proper fractions 1/100 and 99/100 and the decimal equivalent of every proper fraction between these two whose denominator is not greater than 100. These decimal expressions are arranged in order of magnitude. There seems to be no doubt that this is the form of Brocot's original table. In the preface to the first German edition (Berlin, 1871, xvi, 52 p.), which I have not seen, it is stated that this table was wholly recalculated. The next edition of this table of 1879 was in the monograph

W. H. Rasche, *Gear Train Design. A new method of Using Brocot's Table of Decimal Equivalents in Calculating the Numbers of Teeth in a Gear Train* (Virginia Polytechnic Institute, *Engineering Experiment Station Series, Bulletin*, no. 2, June 1926) 111 p. 15.4×23.8 cm. A second edition, with a revision of the introductory text only, appeared in this series as *Bulletin*, no. 14, March, 1933. In both editions, the table occupies pages 71–111. I am indebted to Professor Rasche for kindly supplying me not only with a copy of his *Bulletin* 14, but also with an incomplete copy of the German work of 1879.

After Brocot's table of 1862 no addition was made to it for 73 years. Then came

Earle Buckingham, *Manual of Gear Design Section One. Eight Place Tables of Angular Functions in Degrees and Hundredths of a Degree and Tables of Involute Functions, Radians, Gear Ratios, and Factors of Numbers*, New York, Machinery, 148 Lafayette St., 1935. 21.5×26 cm. In this volume Professor Buckingham has reduced the Rasche-Brocot Table (p. 147–169) from eleven to eight places of decimals, throughout its range, but he has added a new eight-place table, corresponding to the increase in teeth of a gear from 100 up to 120. As already noted, Page extends this new table to 11 places of decimals. In none of these tables before Page and back to Brocot, was it possible readily to find the decimal equivalent of any particular fraction.

Many references might be given to other works where minor gear-ratio tables may be found. The following two samples will suffice:

Brown & Sharpe Mfg. Co., *Formulas in Gearing with Practical Suggestions*. Seventeenth ed., Providence, R. I., 1942, p. 223–245. 15.1×22.9 cm.

Machinery's Handbook for Machine Shop & Drawing Office . . ., Eleventh ed. New York, The Industrial Press, 1941, p. 671–675, 1080–1103. 11.5×17.9 cm.

But the mathematical consideration of expressing proper fractions decimally was developed many years earlier, by Henry Goodwyn in the following four publications of 1816–1823:

1. [*The First Centenary of a Series of Concise and Useful Tables of all the Complete Decimal Quotients which can arise from dividing a Unit, or any Whole Number less than each Divisor, by all Integers from 1 to 1024*, London,] 1816, xiv, 18 p., 20×25.3 cm.

2. *The First Centenary of a Series of Concise and Useful Tables of all the Complete Decimal Quotients, which can arise from dividing a Unit, or any Whole Number less than each Divisor, by all integers from 1 to 1024. To which is now added A Tabular Series of Complete Decimal Quotients for all the Proper Vulgar Fractions, of which, when in their lowest terms, neither the Numerator, nor the Denominator, is greater than 100: with the Equivalent Vulgar Fractions Prefixed*. London, 1818, xiv, 18, viii, 32 p. 20×25.3 cm. No. 1, without title page, and called by Goodwyn a "specimen" is identical with the first part of no. 2 and all of these pages were printed in 1816. The title page and independently paged second part were added in 1818.

3. *A Tabular Series of Decimal Quotients for all the Proper Vulgar Fractions, of which, when in their lowest Terms, neither the Numerator nor the Denominator is greater than 1000*, London, 1823, v, 153 p. 20×25.3 cm.

4. o *A Table of the Circles arising from the Division of a Unit, or any other Whole Number, by all the Integers from 1 to 1024, being All the pure Decimal Quotients that can arise from this Source*. London, 1823, v, 118 p. To express 1/1021 as a repeating decimal requires a "circle" of 1020 digits.

These publications are excessively rare. There is a copy of no. 1 in the Yale University Library; of no. 2, and a film of no. 3, at Brown University; of no. 3 at the John Crerar Library in Chicago. There is no copy of no. 4 in the principal libraries of America. Nos. 3 and 4 were published anonymously. All four of these tables are described by J. W. L. Glaisher (1) in his *Report of the Committee on Mathematical Tables*, Br. Assoc. Adv. Sci., *Report*, 1873; (2) in his "On circulating decimals with special reference to Henry Goodwyn's *Table of Circles* and *Tabular Series of Decimal Quotients* (London, 1818–1823)," Cambr. Phil. So., *Proc.*, v. 3, p. 185–206.

And, finally, we may give a reference to C. F. Gauss's "Tafel zur Verwandlung gemeiner Brüche mit Nennern aus dem ersten Tausend in Decimalbrüche," in his *Werke*, v. 2, Göttingen, second ed., 1876, p. 411–434. 22.5×27.6 cm. When this table was prepared does not seem to be known; it was found among Gauss's papers after his death in 1855.

For such discussions by Gauss, Goodwyn, and others, see D. H. Lehmer, *Guide to Tables in the Theory of Numbers*, Washington, D. C., National Research Council, 1941.

R. C. A.

88[A, F, P].—*Formulas in Gearing with Practical Suggestions.* Seventeenth ed., Providence, R. I., Brown and Sharpe Mfg. Co., 1942. 266 p. 15.1×22.9 cm.

Brown and Sharpe has long been one of the greatest concerns in this country for manufacturing machines, tools, and instruments involving high precision in workmanship. Indeed, during past years their products have been in international demand. Among various publications of this firm are two anonymous volumes which have passed through many editions. *Practical Treatise on Gearing*, by Oscar James Beale, appeared first in 1886 as a volume of about 130 p.; the twenty-fourth ed. of 1942 contains 244 p. The second volume, by Charles C. Stutz, and the one before us for review, was first published in 1892 as a volume of 69 p. The chief additions and changes were made in the fourth, tenth, and twelfth editions of 1905, 1929, and 1936. The current demand for the work is indicated by the fact that there have been four editions, each of 1000 copies, since the twelfth in 1936. The sixteenth and seventeenth editions both appeared in 1942.

The first quarter of the book is mainly occupied by descriptive matter and formulas, while the remaining pages are filled with tables. The descriptive text deals with such topics as systems of gearing, definitions and classifications applied to gearing and pitch of gears, spur gearing, bevel gears, worm and worm wheel, spiral and screw gearing, epicyclic gearing, gearing of lathes for screw cutting, sprockets, and strength of gears. Among the tables most of the pages (123–245) are taken up with tables used by mathematicians, namely: five-place table of natural sines, cosines, tangents, and cotangents for every minute of the quadrant (reproduced by "courtesy of The International Correspondence Schools, Scranton, Pa."); table of prime numbers and factors 1 to 10,200; table of prime numbers and lowest factors 10,000 to 100,000; gear ratios and their decimal equivalents ("Brocot's table" from Machinery's *Handbook*, various ratios from 1/60 =0.0167 to 59/60=0.9833); six-place logarithms for all gear ratios from 100/24 to 24/100 arranged in numerical order; table of ratios of two gears with their decimal equivalents (100/24 =4.1667 to 24/100=.2400). We have discussed gear ratios at length in RMT **87**.

The tables of prime numbers and factors are almost wholly taken from Edward Hinkley, *Tables of Prime Numbers and Prime Factors of the Composite Numbers from* 1 *to* 100,000, Baltimore, 1853. Many errors in this volume are listed by L. J. Comrie, p. xi–xii of Br. Ass. Adv. Sci., *Mathematical Tables*, v. 5: *Factor Table*, London, 1935. In 1935 Mr. Comrie drew the attention of the Brown & Sharpe Mfg. Co. to errors in its factorization tables, in the eleventh edition (1933), mainly copied from Hinkley, of the following 30 numbers: 2198, 2798, 3632, 4086, 4396, 5506, 6998, 7011, 7160, 7264, 8172, 8285, 8792, 8815, 8844, 8901, 9224, 9542, 9543, 9696, 9788, 9810, 10134, 11747, 16107, 17633, 56323, 58301, 65959, 93617. Corrections of all of these errors have been made in the present edition; indeed, except for 8172, they had been already made in the twelfth edition (1936).

R. C. A.

MATHEMATICAL TABLES—ERRATA

By offering the means for improving available tables it is hoped that the Errata, listed or discussed under this heading from time to time, may serve a very useful purpose. The total unreliability of a table will occasionally be indicated. All readers knowing of errors in tables are invited to send a list of them to the Editor for publication. The Committee desires to become a clearing-house for all information of this kind. In RMT **75** we have already given references to lists of errors in tables by Rhaeticus, 1596 and 1613; and in RMT **88**, to errors in a work by Stutz, 1933.

For convenience in reference the Errata lists will be numbered consecutively.

1. E. Gifford, *Natural Sines to Every Second of Arc and Eight Places of Decimals*, second ed., Manchester, 1926. Corrections of more than one in the eighth place are followed by three "corrections" of unity in the eighth place.

Page	′	″	Correct figure	Page	′	″	Correct figure
3	25	33	3213	264	57	11	6876
16	31	08	4874	272	15	06	0585
25	06	09	4097	277	08	56	4251
39	24	06	9784	286	36	37	7629
47	44	01	6750	292	36	29	0404
52	31	59	7998	303	24	09	4105
54	54	09	5349	303	24	52	7391
67	04	26	7475	305	45	49	4293
71	42	05	1103	307	01	09	5644
77	40	27	0634	309	22	25	3305
84	59	12	9609	319	08	53	8797
91	07	38	6319	327	21	46	2241
91	08	08	0359	333	25	27	7580
91	09	18	3118	333	25	28	7856
93	22	18	7933	333	25	29	8131
100	34	57	9565	335	47	59	7785
110	17	29	4976	336	50	02	1289
110	19	51	0334	337	00	57	9207
113	48	52	0434	338	13	14	8399
115	03	59	6351	338	16	49	6309
118	33	39	0751	340	30	19	3666
126	54	48	5539	343	06	52	5677
133	09	38	0330	345	29	18	8202
142	35	09	2245	346	30	24	5396
142	39	27	6846	348	59	24	5559
155	48	41	1005	350	10	54	2404
156	52	37	3978	355	08	17	0581
160	30	46	9739	357	25	32	6899
165	20	39	3441	359	40	18	4595
165	20	46	6456	359	41	12	7812
166	38	52	3486	359	46	09	0398
171	27	27	0675	359	47	49	4797
185	40	39	0522	365	42	09	9062
186	59	26	9678	365	49	55	9394
202	37	52	4374	367	01	41	5699
204	52	23	5472	367	08	39	3681
216	56	58	7118	369	21	51	8343
217	02	39	0871	369	21	52	8575
224	17	48	4212	370	32	53	1701
239	42	29	7599	371	45	28	5498
254	15	05	8475	372	50	23	3085
258	58	32	8628	372	50	24	3314

Page	′	″	Correct figure	Page	′	″	Correct figure
372	50	25	3543	442	37	56	7261
372	50	26	3772	442	37	57	7397
372	56	58	3301	442	37	58	7534
373	09	05	8496	442	37	59	7671
374	17	00	5837	444	57	33	6501
377	48	35	9392	445	02	22	5123
381	28	57	9804	445	02	25	5523
383	41	27	1553	445	02	28	5923
383	45	42	6278	448	32	38	3488
383	45	43	6493	448	33	09	7492
384	50	06	2790	448	34	28	7686
385	03	14	0595	449	43	39	8396
387	28	23	8273	449	48	09	2793
388	33	28	1896	451	07	33	9192
388	34	42	7302	457	09	31	6172
392	11	16	8798	459	22	13	3889
400	30	46	4898	461	48	29	1100
405	22	19	2193	466	31	42	0292
408	51	02	0362	469	01	37	4527
416	18	35	0400	473	49	27	3699
418	30	14	9596	481	02	44	4551
418	36	48	6308	485	44	25	6908
426	53	12	7274	485	49	03	8505
427	08	43	4104	487	09	38	2283
427	09	45	3814	495	20	14	7003
429	20	57	8505	497	48	37	3717
435	20	34	8821	498	51	25	3878
435	21	52	0285	499	02	12	2394
436	31	33	5244	499	05	59	5676
436	37	59	1269	499	08	03	2881
437	41	49	4494	104	15	19	2955
438	52	51	9460	201	20	15	6973
440	12	03	2370	413	49	06	3946
441	25	27	4299				

W.P.A. Project No. OP 165-2-23-1250 at Philadelphia, Pa., 1941, under the sponsorship of the U. S. Coast and Geodetic Survey, L. G. Simmons, Senior Geodetic Engineer, in charge. Compare RMT **77**.

I have checked all of the errors here listed as well as 1126 other eighth-place unit errors found by the Coast and Geodetic Survey, with the copy of the second edition of Gifford's *Natural Sines* in the Library of Brown University. The result was to find that the following five listed errors were not errors at all:

Page	′	″	Survey	Gifford
303	24	52	7391	7391
374	17	00	5837	5837
405	22	19	2193	2193
440	12	03	2370	2370
466	31	42	0292	0292

All of the entries listed in the Errata above (+1126) were also checked with the *Eight-figure Table of the Trigonometrical Functions* by J. T. Peters. In this way six cases were found in which the "Correct figures" of the Coast and Geodetic Survey were called into question. These are as follows:

Page	°	′	″	Survey	Gifford	Peters
104	17	15	19	2955	2954	2954
201	33	20	15	6973	6974	6974
367	61	01	41	5699	5799	5700
413	68	49	06	3946	3947	3947
427	71	09	45	3814	3184	3813
445	74	02	28	5923	5823	5922

R. C. A.

End-figures are missing sin 1°44′41″ and 42″, namely: 0 and 5 respectively.

L.J.C.

Sin 36°, *for* 0.587 78255, *read* 0.587 78525.

F. W. Hoffman, 689 East Ave., Pawtucket, R. I.

2. H. BRIGGS, *Trigonometria Britannica*, trigonometry by H. GELLIBRAND, Gouda, 1633.

	for	read
tan 6°.24.	0.10934 01888	0.10934 11888
tan 35°$\frac{100}{100}$	0.72654 45280	0.72654 25280; tan 36° is correct
sin 64°.49	0.90221 . . .	0.90251 . . .

J. T. PETERS, *Siebenstellige Werte der trigonometrischen Funktionen von Tausendstel zu Tausendstel des Grades*, Leipzig, 1918, p. [iii]; also English edition, New York, 1942, p. [iii].

tan 19°.29 *for* 0.35099,90945 *read* 0.34999 90945

tan 77°.34 to 77°.67, 34 entries where the first digit should be 4 instead of 9.

AMELIA DE LELLA, *Five Place Table of Natural Trigonometric Functions to Hundredths of a Degree*, New York, 1934, Preface.

3. A. J. C. CUNNINGHAM, *Binomial Factorisations*, v. 1, London, 1923; see UMT **1**.

P. 243, in the table for which $(1/10)(n^2+1)$ is a prime, col. 11, *for* 4683 *read* 9683.

P. 244, in the table for which $(1/13)(n^2+1)$ is a prime, *omit* the entry 671; and *for* 3930 in col. 4, *read* 2930.

L. EULER, "De numeris primis valde magnis," 1764; see UMT **1**.

193 is given as a divisor of 82^2+1, whereas it divides 81^2+1; 1068^2+1 is said to be equal to $5^6 \cdot 73$, instead of the correct factorization $5^6 \cdot 73$; 1080^2+1 is said to have 773 as a factor, whereas 773 divides 1090^2+1, and 1080^2+1 is a prime. These errors remain uncorrected in the later editions of this paper, even in Euler's *Opera Omnia*, s. 1, v. 3, 1917.

J. W. WRENCH, JR.

4. L. J. COMRIE, editor, *Barlow's Tables . . .*, fourth ed., London, 1941. See RMT **82**.

P. 5, cube root of 197, *for* 5.8186497, *read* 5.8186479.

In the first edition, 1930, p. 25, the difference following $\sqrt{10n}$ for 1156 should be 46494, not 45494.

L.J.C.

UNPUBLISHED MATHEMATICAL TABLES

The list of unpublished mathematical tables, on which we shall later make report, is long, but this list can doubtless be greatly extended. We hope that anyone knowing of such tables in public or private hands, will acquaint us with the facts. The Committee desires to become a clearing-house for all information of this kind. It believes that the dissemination of such information is highly desirable, and may render notable services.

1. JOHN WILLIAM WRENCH, JR. (1911–) *Complete factorization of integers of the form of* n^2+1 *for* $1 \leqq n \leqq 16{,}200$. Ms. in possession of Dr. Wrench, and a film copy in the Library at Brown University.

There is also a portion of the table, $n \leqq 10{,}000$, in the Library of Yale University where the table was part of a doctoral dissertation (1938).

The first table of this kind was given by L. Euler, "De numeris primis valde magnis," Acad. Sci. Petrop., *Novi Commentarii*, v. 9 (1762–3), 1764, p. 112–117. (For 3 other editions see D. H. Lehmer, *Guide to Tables in the Theory of Numbers*, National Research Council, 1941.) The range of

this table is $1 \leq n \leq 1500$ and the largest prime is purposely omitted in the majority of cases; hence a prime is indicated, for the most part, by a blank space. Gauss gave a factor table of n^2+1 (*Werke*, Göttingen, v. 2, second ed., 1876, p. 478–481), where the numbers are selected according to the criterion that the largest prime factor should not exceed 197. He has given an enumeration of 657 such integers (from $n=2$ to $n=14\ 033\ 378\ 718$) with their complete factorizations—the factor 2 being omitted throughout. Gauss constructed similar factor tables for integers of the form n^2+k^2, where k assumes the values 2, 3, 4, . . . , 9, inclusive.

In his *Binomial Factorisations*, v. 1, London, 1923, p. xxvi, the late A. J. C. CUNNINGHAM mentions an unpublished ms. table of his which gives the complete factorization of n^2+1 for $1 \leq n \leq 15{,}000$. On the basis of this table he enumerates (p. 238–239) the positive values of $n < 15{,}000$ for which n^2+1 is prime. (I have discovered no errata in his list.) He has also tabulated (p. 240–244) in separate groups those values of n for which $(1/2)(n^2+1)$, $(1/5)(n^2+1)$, $(1/10)(n^2+1)$, $(1/13)(n^2+1)$, $(1/17)(n^2+1)$, are prime. For errors in these, and in Euler's table, see MTE 3.

As Gauss first indicated (*Werke*, v. 2, second ed., p. 497–500), one of the most important uses of such a table is the derivation of arctangent relations for the calculation of π. I have elaborated this theme in "On the derivation of arctangent equalities," *Amer. Math. Mo.*, v. 45 (1938), p. 108–109. In the same volume, in an expository article "On arccotangent relations for π" (p. 657–664) D. H. Lehmer compared the relations with regard to ease in application to the calculation of π to a large number of decimal places.

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MECHANICAL AIDS TO COMPUTATION

1. *Seventeenth Century Calculating Machines.*—In *Nature*, v. 150, 31 Oct. 1942, p. 508–509, is an address delivered at a memorial luncheon held in London on 19 October, by the president of the Royal Astronomical Society, S. Chapman, "Blaise Pascal (1623–1662) tercentenary of the calculating machine." A report of this luncheon organized by a small committee, of which L. J. Comrie was chairman, appeared in this same issue of *Nature*, p. 527, "Pascal tercentenary celebration." The 120 guests included many distinguished French scientists, as well as an official deputation from General de Gaulle's headquarters. At the age of nineteen Pascal invented the first calculating machine, made in 1642. By this means he hoped to assist his father, Etienne Pascal (d. 1651), discoverer of the limaçon, in statistical work involving additions and subtractions of sums of money. Such operations were those to which its applications were confined. During the following decade he made improvements, and one of his machines of 1652, bearing his signature, is preserved in the Conservatoire des Arts et Métiers. A replica of this is in the Science Museum, South Kensington, London. A detailed account of this machine by Denis Diderot (1713–1784), with illustrations, was published in *Encyclopédie, ou Dictionnaire Raisonné des Sciences des Arts et des Métiers*, Paris, v. 1, 1751, p. 680–684, Planches, v. 5, 1767, algèbre et trigonométrie, plate II. Also in 3rd ed., Geneva, and Neuchâtel, 1779, v. 3, p. 381–388, plates, v. 1. See also *Encyclopédie Méthodique. Mathématiques*, . . . , Paris, v. 1, 1784, p. 136–142; and *Oeuvres Complètes de Blaise Pascal*, v. 2, Paris, Hachette, 1860, p. 368–380. Chapman notes, "In 1652 he [Pascal] presented one of the last of his fifty models, with a famous letter, to Queen Christina of Sweden. He also wrote a prospectus of his invention that would do credit to a modern school of salesmanship." This

letter and prospectus are given on p. 359–368 of the volume of the *Oeuvres* here cited. The account of the "Machine Arithmétique de M. Pascal" in *Machines et Inventions approuvées, par l'Académie Royale des Sciences, depuis son Établissement jusqu'à present*, v. 4, Paris, 1735, p. 137–140 is accompanied by two fine plates. An account of Pascal's machine is also given in M. d'Ocagne, *Le Calcul Simplifié par les Procédés Mécaniques et Graphiques*, second ed., Paris, 1905, p. 24–29.

In the luncheon exhibition room were replicas of Pascal's machine, and also of one of the first three English calculating machines made in 1663–1666 by the diplomatist, mathematician, and inventor, Sir Samuel Morland (1625–1695). Morland's Trigonometrical Calculating Machine was invented in 1663 and constructed in 1664. It "provides a means of rapidly constructing triangles to scale from given data by using graduated bars and circles. Any ordinary problem in plane trigonometry which can be solved by plotting on paper may be quickly solved by this instrument. The sin, cos, tan, etc., of any angle may be read off at once. Multiplication and division may also be performed by employing the graphical method of similar triangles." Morland constructed also another type of multiplying instrument, as well as a calculating machine which he invented in 1666 and which he described as a "new and most useful instrument for addition and subtraction of pounds, shillings, pence, and farthings; without charging the memory, disturbing the mind, or exposing the operator to any uncertainty which no method hitherto published can justly pretend to." See D. Baxandall, *Catalogue of the Collections in the Science Museum, South Kensington . . . Mathematics*, I. *Calculating Machines and Instruments*, London, 1926, p. 8, 14–16, and plate.

In 1671 G. W. Leibniz (1646–1716) conceived the idea of a calculating machine which would perform multiplication by rapidly repeated addition. It was not until 1694 that his first complete machine was actually constructed. It was preserved in the former Royal Library at Hannover. Leibniz describes his machine in "Brevis descriptio machinae arithmeticae," *Miscellanea Berolinensia*, v. 1, 1710, p. 317–319, with a plate exhibiting the appearance of the instrument. Cf. Louis Couturat, *La Logique de Leibniz, d'après des documents inédits*, Paris, 1901, p. 115–116, 295–296.

Attention may be drawn to interesting articles on Calculating Machines by David Baxandall in *Encyclopædia Britannica*, fourteenth ed., v. 4, 1929, p. 548–553; and by E. M. Horsburgh in (a) *A Dictionary of Applied Physics*, ed. by R. Glazebrook, v. 3, London, 1923, p. 193–201, and (b) *Modern Instruments and Methods of Calculation. A Handbook of the Napier Tercentenary Exhibition*, London, 1914, p. 69–277. See also Antonino Asta's "Calcolatrici Macchine," *Enciclopedia Italiana*, Milan and Rome, v. 8, 1930, p. 352–358.

R. C. A.

NOTES

1. *Who Was Who 1929–1940, a Companion to Who's Who*, London, 1941, contains sketches of two notable table makers, JOHN ROBINSON AIREY (1868–1937) and ERNEST WILLIAM BROWN (1866–1938). Mr. Airey was principal of the West Ham Technical Institute, 1912–18, and principal of the City of Leeds Training College, 1918–33. He became a member of the Mathematical Tables Committee of the Br. Ass. Adv. Sci. in 1911, and was its secretary 1916–29. He was a coeditor of the *Philosophical Magazine*. Here, in *Proceedings* of the London Physical So., and in *Reports* of the Br. Ass. Adv. Sci., he published many of his tables. His brother Sir Edwin Airey is head of a large firm of building contractors and engineers. See *Times*, London, 17 Sept. 1937; see RMT **81.**

Professor Brown was a sixth wrangler at Cambridge in 1887, and came to the United States in 1891, as an instructor in mathematics at Haverford College. His connection with Yale University began in 1907 as professor of mathematics. Shortly after the publication of his great *Tables of the Motion of the Moon*, 1919, he became Sterling professor of mathematics, 1921–31; then first Josiah Willard Gibbs professor, 1931–32; and professor emeritus, 1932–38. He was naturalized as a citizen of the United States in 1922. His outstanding position as mathematician and mathematical astronomer, and the long list of honors conferred on him, are set forth, with a complete bibliography of his writings, and a portrait, in R. C. Archibald, *A Semicentennial History of the American Mathematical Society 1888–1938*, New York, 1938. See also C. G. Darwin, *Obituary Notices of Fellows of The Royal Society*, 1940, p. 19–22+portrait plate; W. J. Eckert, *Pop. Astr.*, v. 47, 1939, p. 63–66, portrait; and F. Schlesinger, *Astrophysical Jn.*, v. 89, 1938, p. 152–155 and portrait.

2. In London Math. So., *Jn.*, v. 16, 1941, p. 139–144 is an obituary notice by E. T. Whittaker of another notable table maker EDWARD LINDSAY INCE (1891–1941). He was a professor of pure mathematics at the Egyptian University, Cairo, 1926–31; a lecturer at Imperial College, South Kensington, 1932–35; and head of the department of Technical Mathematics, University of Edinburgh, 1935–41. After eight years devoted to the task he published (1932) tables of the characteristic numbers of Mathieu's functions, with their zeros and turning points. Volume 4, of the *Mathematical Tables* of the Br. Ass. Adv. Sci., was Ince's *Cycles of Reduced Ideals in Quadratic Fields*, 1934. See RMT **22, 41.**

3. In *Science*, n.s., v. 96, p. 294–296, 25 Sept. 1942, there is an account by R. C. Archibald of the set-up and remarkable achievements of "THE NEW YORK MATHEMATICAL TABLES PROJECT" which has been in existence for just five years, as a part of the Government's Works Projects Administration Program. Lyman J. Briggs, director of the U. S. Bureau of Standards, has been determining the Project's policies and activities, and overseeing the distribution of its publications; and Arnold N. Lowan has been the Project's capable technical supervisor. A year ago 350 computers were working, in two shifts, in order fully to utilize 150 computing machines. By August the number of

computers had been reduced to 250. Since the WPA is to be liquidated on 1 February 1943 the New York Mathematical Tables Project is also to be terminated by that date. Had all the computers in the Project been dispersed, the loss to scientific research would have been irreparable. We learn, however, that 47 members of the group have been taken over by the U. S. Bureau of Standards and by the Hydrographic Office.

Unless a much more adequate nucleus of the present Project is preserved, with calculating machines, a large number of mathematical tables now in process may never come to publication.

R. C. A.

QUERIES

1. Tables to Many Places of Decimals.—Various tables, computed to from 15 to 60 or more places of decimals, have been published in the distant past, and continue to appear in recent times; several tables of this kind are referred to in this issue of *MTAC*. In what specific problems of research do their solutions make such tables highly desirable? There are doubtless many problems of this kind which might be formulated by the specialists in different fields. An appeal for contributions of such formulations is herewith made. Professor Lehmer, a specialist in the theory of numbers, makes an interesting start in this direction in QR 1, extracted from a personal letter dated 7 November 1942.

R. C. A.

QUERIES—REPLIES

1. Tables to Many Places of Decimals.—As to the sort of problems in which great accuracy in logarithms is helpful, you cannot draw the line between 20 figure problems and over 100 digit problems. I could not pretend to describe all such problems but those with which I have had something to do are of the following type: There is defined some numerical function whose value is an integer for each integer value of the variable. Quite often the function is defined by words (such as the number of ways that such and such a thing can be done). It often happens that when the argument of the function is only fairly large the value of the function is extravagantly large. Even so, this value is an integer and it is either even or odd (or perhaps we should like to know if it is divisible by 23 to check up on a conjecture). Perhaps there is no workable formula with which to calculate an isolated value of the function. Perhaps even a recursion formula for calculating a complete table of the function is not feasible. Then we may turn to an approximate formula, a series consisting of infinitely many terms which are values of an analytic function [often a Bessel function]. If enough terms of this series are taken so that the sum of the remainder terms is less than 1/2 in absolute value then the integer we are looking for is the nearest integer to the value we get from the series. Since these terms must add up to an extravagantly large amount some terms must be calculated with extreme accuracy. We do not have tables of Bessel functions to say 100

decimal places, so we have to use an asymptotic method involving exponentials or (what is the same thing) logarithms. A fairly workable table of logarithms to a great many decimals thus is instrumental in calculating a value of a numerical function which may be correct to only one or two decimal places but is nevertheless correct to something like 100 significant figures.

Here is another interesting example. Let D be a positive integer; then there is a remarkable connection between the fractional part of $\exp(\pi\sqrt{D})$ and the number of classes of binary quadratic forms of determinant $-D$. When the number of classes is quite small this fractional part may be very close to zero or one. For example for $D = 163$ we have

$$e^{\pi\sqrt{163}} = 262537412640768743.99999999999925007259 7 \cdots .$$

To discover just how close to an integer this exponential is requires about 35 decimal place accuracy even if one is not interested in just that integer to which it is so close.[1] There is reason to believe that such remarkable approximations to integers by exponentials of this type do not persist. Experiments on such questions manifestly require exceedingly accurate tables of logarithms.

D. H. L.

[1] This integer is really very interesting since it is $744 + (2^6 \cdot 3 \cdot 5 \cdot 23 \cdot 29)^3$ where 744 is the constant coefficient in the Fourier series development of Klein's celebrated absolute modular invariant.

RMT 81—Supplementary Note.

Through an oversight a review was not included in this issue of PROJECT FOR COMPUTATION OF MATHEMATICAL TABLES, *Tables of Circular and Hyperbolic Sines and Cosines for Radian Arguments*. Prepared by the Federal Works Agency, Work Projects Administration for the State of New York, conducted under the sponsorship of the National Bureau of Standards. New York, 1940, xx, 405 p. A detailed review will appear in our next issue. We simply note here that the volume includes tables of sin x and cos x for $x = [0.0000(0.0001)1.9999; 9D]$, and for $x = [0.0(0.1)10.0; 9D]$.

CLASSIFICATION OF TABLES, AND SUBCOMMITTEES

A. Arithmetical Tables. Mathematical Constants
B. Tables of Powers
C. Logarithms
D. Circular Functions
E. Hyperbolic and Exponential Functions
Professor DAVIS, *chairman*, Professor ELDER
Professor KETCHUM, Doctor LOWAN

F. Theory of Numbers
Professor LEHMER

G. Higher Algebra
Professor LEHMER

H. Tables for the Numerical Solution of Equations
J. Summation of Series

I. Tables connected with Finite Differences
K. Statistical Tables
Professor WILKS, *chairman*, Professor COCHRAN, Professor CRAIG
Professor EISENHART, Doctor SHEWHART

L. Higher Mathematical Functions
M. Integral Tables
Professor BATEMAN

N. Interest and Investment
O. Actuarial Tables
Mister ELSTON, *chairman*, Mister THOMPSON, Mister WILLIAMSON

P. Tables Relating to Engineering

Q. Astronomical Tables
Doctor ECKERT, *chairman*, Doctor GOLDBERG, Miss KRAMPE

R. Geodetic Tables
S. Physical Tables
T. Critical Tables of Chemistry
U. Navigation Tables

Z. Calculating Machines and Mechanical Computation
Doctor COMRIE, *chairman*, Professor CALDWELL, *vice-chairman*
Professor LEHMER, Doctor MILLER, Doctor STIBITZ, Professor TRAVIS

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